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LETTER TO THE EDITORS

Comments on "Transition from transpiration to film cooling"

In a paper Eckert and Cho [1] reported an analysis of the transition from transpiration to film cooling, dedicated to the seventieth birthday of J. P. Hartnett. First, I would like to take this opportunity to congratulate Professor Hartnett on his birthday. Second, with regard to ref. [1] I would like to add three comments to the section named: Ideal transpiration cooling.

1. The minus sign in equation (1) is erroneous. The heat transfer from hot fluid to wall should read:

$$q_w = k\frac{dT}{dy}\bigg|_{y=0}. \qquad (1)$$

This mistake has no consequences in the continuation of the analysis.

2. In Fig. 2 the results of experiments and empirical correlations are presented. An alternative way of assessing the effect of blowing on heat transfer is offered by the classical film model. This model is based on a one-dimensional analysis of a stagnant film through which heat and mass are transferred between a fluid and a wall. Correction factors for the effect of condensation/evaporation on heat transfer have been derived by Colburn and Drew [2] and Ackermann [3]. An equivalent correction factor for the effect of suction/blowing has been derived by Mickley *et al.* [4]. The thermal correction factor, often called the Ackermann correction, predicts the ratio of heat transfer with mass transfer and without mass transfer.

In the notation of ref. [1] it can be expressed as:

$$\frac{St_w}{St_0} = \Theta(\phi_t) = \frac{-\phi_t}{e^{-\phi_t}-1}. \qquad (2)$$

To apply the correction function Θ, the dimensionless mass flux ϕ_t, defined as:

$$\phi_t = -\frac{\rho v_w c_p \delta_t}{k}, \qquad (3)$$

with:

$$\delta_t = \frac{k(T_s - T_w)}{q_{w0}}, \qquad (4)$$

has to be expressed in the variables used in ref. [1]. From equations (1), (3) and (4) and equations (2) and (3) of ref. [1] it follows that:

$$\phi_t = -\frac{M}{St_0}. \qquad (5)$$

Now it is possible to compute St_w/St_0 for various M/St_0. In Table 1 some results are presented.

The tabulated values of St_w/St_0 and M/St_0 can be compared directly with the results drawn in Fig. 2 of ref. [1]. One can readily see the good agreement of the film model values and experimental results. Furthermore, experiments and models reveal that heat transfer between the hot fluid and wall tends to zero for large blowing rates. This reduced heat transfer is predicted better by the film model than by the drawn empirical relations. Hence, the film model is recommended for future engineering computations.

3. Eckert and Cho [1] mention a procedure for the design of transpiration cooling arrangement. To determine the wall temperature T_w an energy balance at the interface of porous wall and hot fluid ($y = 0$), see Fig. 1, is proposed.

The heat flux from the hot fluid to the interface follows from equation (2) of ref. [1]. For the heat transported away from the interface (through the porous wall) equation (5) of ref. [1] is put forward. This latter expression, however, is not correct, as will be explained below.

In the fluid-saturated porous wall, heat is transported by conduction and convection. The energy equation of this system and its analytical solution follow from Brouwers [5] as:

$$T(Y) = (T_w - T_c)\left(\frac{e^{-(Y\phi_p/\delta_p)}-1}{e^{-\phi_p}-1}\right) + T_c. \qquad (6)$$

The heat transfer through the interface of hot fluid/porous medium, i.e. at $Y = \delta_p$, then reads:

$$q_w = \frac{k_p}{\delta_p}[\Theta(\phi_p) - \phi_p](T_w - T_c). \qquad (7)$$

In equations (6) and (7) the dimensionless thermal mass flux through the porous wall is introduced, defined as:

$$\phi_p = -\frac{\rho v_w c_p \delta_p}{k_p}, \qquad (8)$$

where k_p represents the effective thermal conductivity of fluid-saturated porous wall. One can readily see that equation (5) of ref. [1] is not in agreement with equation (7).

In the special case only of large negative ϕ_p, equation (7) tends to equation (5) of ref. [1]. Equation (2) namely reveals that for large negative ϕ_p the function $\Theta(\phi_p)$ tends to zero. However, as discussed in the previous comment, in this limiting case heat transfer is governed by the heat resistance between wall and hot fluid, implying that the porous wall is isothermal ($T_w = T_c$). Equation (5) of ref. [1] is therefore useful only if: T_w (instead of T_s) is imposed *and* v_w is very large. Here, however, T_w follows from an energy balance and the blowing velocity v_w is imposed and its magnitude finite. Hence, equation (5) of ref. [1] cannot be used for assessing T_w.

To determine T_w appropriately, equation (2) of ref. [1] and equation (7) are equated. If the heat transfer from the hot fluid–wall is described with the help of the film model:

$$q_w = St_0 \rho c_p u_s \Theta(\phi_t)(T_s - T_w) \qquad (9)$$

which is allowed as pointed out in the previous comment, then the energy balance yields:

$$\frac{T_s - T_w}{T_w - T_c} = e^{-\phi_p}\left[\frac{e^{-\phi_t}-1}{e^{-\phi_p}-1}\right]. \qquad (10)$$

Table 1. Effect of blowing on heat transfer according to film model

M/St_0	St_w/St_0
0	1
2	0.31
4	0.074
6	0.015

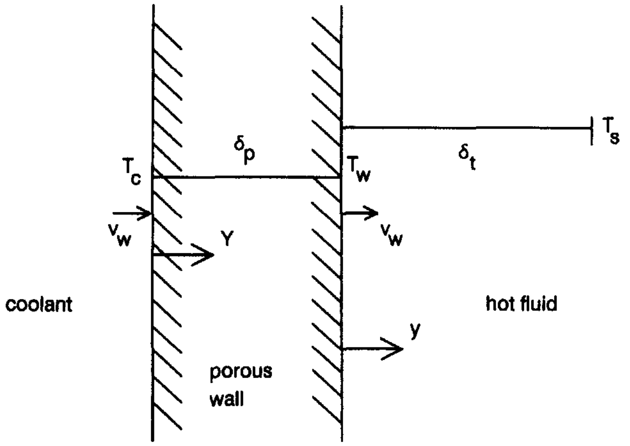

Fig. 1. Schematic representation of hot fluid next to porous wall.

Via this equation T_w can be expressed explicitly in terms of T_s, T_w and the dimensionless transpiration rates ϕ_t and ϕ_p. Equation (10) confirms that for large negative ϕ_t and ϕ_p (large positive v_w) T_w tends to T_c and, consequently, the porous wall is isothermal. Equation (10) forms an analytical and compact relation which is suited to engineering end purposes.

The above-mentioned comments state the useful and powerful film model approach to transpiration cooling. The comments will hopefully contribute to a better understanding and smoother dimensioning of transpiration cooled equipment.

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