expression (18). The same conclusion as for the condensation analysis can therefore be drawn with regard to multiple regions in the film.

4. CONCLUSION

In the present study the possible existence of multiple superheated and saturated regions in a stagnant binary film, consisting of a vapour and non-condensables, has been examined. The analysis yields the important conclusion that, once fog formation has been detected with the slope condition and a saturated and superheated region determined with the tangency condition, the uniqueness of both regions is assured.

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The film model applied to free convection over a vertical plate with blowing or suction

H. J. H. Brouwers†

Akzo Research Laboratories Arnhem, Fibers Division, Department of Mechanical Engineering, Velperweg 76, 6824 BM Arnhem, The Netherlands

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INTRODUCTION

For many years now classical film model correction factors have been successfully used to predict the effect of mass transfer towards a wall on transport phenomena, such as exerted friction and heat and mass transferred. The correction factors can be derived from a stagnant film analysis and applied to systems using either an imposed (transpiration) mass flux or a diffusional vapour flux (by condensation or evaporation). Recent reviews of the film model are found in Bannwart [1], Bannwart and Bontemps [2], Brouwers and Chesters [3] and Brouwers [4]. Whereas the former two authors extended the model to include the effect of mass transfer on film thickness, the latter investigators added fog formation to the model.

The film model expressions have been applied to forced convective heat flow in the presence of an imposed mass flux by Mickley *et al.* [5] and Wang and Tu [6]. Colburn and Drew [7] and Webb and Sardesai [8], among others, fruitfully applied the film model to forced convective diffusional mass transfer. Heat transfer of forced pure vapour flow with wall condensation has been treated with the film model by Mizushina *et al.* [9].

With respect to free convective flow with mass transfer, the film model has been utilized only by Corradini [10] and Vernier and Solignac [11] in an attempt to model the postulated loss-of-coolant accident in a nuclear reactor. Turbulent free convective flow of wall condensing water vapour in air was considered, but poor agreement was found with the experiments performed. Until now, however, the film model predictions have never been applied to free convective flow problems with imposed wall transpiration. For example, fluid injection is an effective way of cooling and reducing heat transfer to surfaces in extremely hot surroundings.

Hence, in this technical note the classical film model is applied to free convective heat transfer with an imposed mass flux. Subsequently, the predictions are compared with the theoretical results of previous investigators. These results are based on an analysis of the governing equations of laminar free convective boundary layer flow over a permeable vertical plate with wall transpiration. In this paper the comparison is restricted to laminar free convection because, to the author's knowledge, data on turbulent free convective flow with wall suction or injection are not yet available.

† Present address: Department of Civil Engineering, Twente University, P.O. Box 217, 7500 AE Enschede, The Netherlands.

FILM MODEL

According to film theory the actual local Nusselt number, denoted by Nu_x, in the presence of mass transfer follows from multiplying the zero suction (or neutral) Nusselt number, Nu_x^*, by a correction factor

$$Nu_x = \Theta_{t,\mathrm{film}} Nu_x^*. \tag{1}$$

The thermal correction factor, commonly referred to as Ackermann correction, follows from Brouwers [4]—among others—as

$$\Theta_{t,\mathrm{film}} = \frac{-\phi_t}{e^{-\phi_t}-1} \tag{2}$$

where the dimensionless mass flux towards the wall reads

$$\phi_t = \frac{\dot{m}c_p}{h_x^*} = -\frac{v_w \rho c_p}{h_x^*}. \tag{3}$$

In this equation h_x^* represents the local heat transfer coefficient in the case of zero mass transfer. For free convective heat transfer it is defined as

$$h_x^* = \frac{Nu_x^* k}{x} \tag{4}$$

where x is a coordinate along the plate. For free convection over an isothermal impermeable vertical plate by thermal buoyancy the local neutral Nusselt number (see Ostrach [12]) reads

$$Nu_x^* = \psi(Pr)\left(\frac{Gr_x}{4}\right)^{1/4}. \tag{5}$$

In Table 1 values of ψ are listed for various Prandtl numbers, taken from ref. [12]. In the next section the film model predictions are extensively compared with the results of previous investigators.

NOMENCLATURE

c_p	specific heat [J kg^{-1} K^{-1}]
E	relative difference between literature and film model, equation (8)
f	dimensionless stream function
Gr_x	Grashof number, $g(t_w - t_\infty)x^3/v^2T_\infty$
g	gravitational acceleration [m s^{-2}]
h_L	mean heat transfer coefficient [W m^{-2} K^{-1}]
h_x	local heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
L	plate length [m]
$\dot{m}$	mass flux to wall [kg m^{-2} s^{-1}]
Nu_x	Nusselt number, $h_x x/k$
n	power-law coefficient
Pr	Prandtl number, $\eta c_p/k$
Q	dimensionless heat flow
T	absolute temperature [K]
t	temperature [°C]
u	component of velocity in the x-direction [m s^{-1}]
v	component of velocity in the y-direction [m s^{-1}]
x	coordinate along the plate [m]
y	coordinate normal to the plate [m].

Greek symbols

η	dynamic viscosity [Pa s]
Θ_t	thermal correction factor for the effect of transpiration
$\Theta'(0)$	dimensionless temperature gradient
v	kinematic viscosity [m^2 s^{-1}]
ξ	non-dimensional injection parameter, equation (11)
ρ	density [kg m^{-3}]
ϕ_t	dimensionless wall mass flux, equation (3).

Subscripts

Eich	pertaining to Eichhorn [13]
film	pertaining to film model
Merk	pertaining to Merkin [14]
Pari	pertaining to Parikh *et al.* [15]
w	wall
∞	ambient fluid.

Superscript

*	pertaining to zero suction.

Table 1. Values of ψ for various Pr according to Ostrach [12]

Pr	0.01	0.72	0.733	1	10	100	1000
ψ	0.0812	0.5046	0.5080	0.5671	1.1694	2.191	3.966

COMPARISON WITH PREVIOUS WORK

The film model correction for the effect of mass transfer on heat transfer is compared with the results of Eichhorn [13], Merkin [14] and Parikh *et al.* [15]. All these foregoing studies concern a laminar free convective boundary layer flow over a vertical plate with imposed transpiration. For the sake of completeness the studies of Sparrow and Cess [16], Mabuchi [17] and Clarke [18] should also be mentioned.

Eichhorn [13] was the first to study the effect of suction and injection on free convective flow. Power-law variations of wall temperature ($t_w - t_\infty = C_1 x^n$) and transpiration velocity ($v_w = C_2 x^{(n-1)/4}$) were considered under which self-similar solutions of the governing equations are possible. For various wall stream function values, f_w, the dimensionless temperature gradient at the wall, referred to as $-\Theta'(0)$, was determined. The Nusselt number ratio now readily follows as

$$\frac{Nu_x}{Nu_x^*} = \frac{\Theta'(0)}{\Theta'(0)^*} = \Theta_{t,\mathrm{Eich}}, \tag{6}$$

which can be compared with the film model correction factor. To this end, the dimensionless mass flux ϕ_t has to be assessed. This property is related to f_w by

$$\phi_t = -\frac{f_w(n+3)Pr}{\Theta'(0)^*} \tag{7}$$

(see equations (3) and (4), and equations (11) and (13) from ref. [13]). For $n = 0$ and $Pr = 0.73$ values of $\Theta'(0)$ were computed by Eichhorn [13] as a function of f_w. These values are summarized in Table 2. A glance at this table shows that $-\Theta'(0)^*$ amounts to 0.507, which is in agreement with Table 1 ($\psi(Pr = 0.733) = 0.508$). Table 2 includes both the pertaining film model correction factors and the relative error defined as

$$E_{\mathrm{Eich}} = \frac{\Theta_{t,\mathrm{film}} - \Theta_{t,\mathrm{Eich}}}{\Theta_{t,\mathrm{Eich}}}. \tag{8}$$

The tabulated values of E_{Eich} indicate that for suction, i.e. $f_w > 0$ and $\phi_t > 0$, the film model predictions correspond well to the numerical results. For large injection rates, however, the deviation increases; E_{Eich} then approaches a value of three. The film model deviates less than 5% from Eichhorn's values for $-0.864 \leqslant \phi_t \leqslant 3.320$.

Merkin [14] solved the governing equations of the process considered at constant wall temperature and uniform suction and injection. In this paper asymptotic solutions were also derived for large suction and injection rates.

For various absolute values of the dimensionless transpiration rate ξ the non-dimensional heat transfer was computed and drawn

$$Q = -\frac{Nu_x v}{x v_w}. \tag{9}$$

Combining equations (3), (4) and (9) yields

$$\frac{Nu_x}{Nu_x^*} = \frac{\phi_t Q}{Pr} = \Theta_{t,\mathrm{Merk}}. \tag{10}$$

The dimensionless mass flux ϕ_t is expressed in ξ by considering

$$\xi = \frac{v_w x}{v}\left(\frac{Gr_x}{4}\right)^{-1/4} \tag{11}$$

and equations (3)–(5)

$$\phi_t = -\frac{\xi Pr}{\psi(Pr)}. \tag{12}$$

In Fig. 1 the resulting ϕ_t and $\Theta_{t,\mathrm{Merk}}$ are depicted for $Pr = 1$ and $\psi(Pr) = 0.5671$ (see Table 1). Also in Fig. 1 the film model correction is drawn for $|\phi_t| \leqslant 3.5$. This figure illustrates the good agreement between the numerical results of Merkin [14] and the basic film model.

Parikh *et al.* [15] studied both numerically and experimentally the problem of free convection over an isothermal porous vertical plate with uniform transpiration. The

Table 2. Numerical results of Eichhorn [13] and the film model ($n = 0$, $Pr = 0.73$)

Eichhorn [13] f_w	$-\Theta'(0)$	Equation (6) $\Theta_{t,Eich}$	Equation (7) ϕ_t	Equation (2) $\Theta_{t,film}$	Equation (8) E_{Eich} (%)
−1.0	0.00748	0.0148	−4.320	0.0582	293.2
−0.8	0.0264	0.0521	−3.456	0.113	116.9
−0.6	0.0725	0.143	−2.592	0.210	46.9
−0.4	0.162	0.320	−1.728	0.373	16.6
−0.2	0.305	0.602	−0.864	0.629	4.5
−0.1	0.399	0.787	−0.432	0.800	1.7
0.0	0.507	1.000	0.000	1.000	0.0
0.1	0.629	1.241	0.432	1.232	−0.7
0.2	0.764	1.507	0.864	1.493	−0.9
0.3	0.912	1.799	1.296	1.784	−0.8
0.4	1.072	2.114	1.728	2.101	−0.6
0.5	1.241	2.448	2.160	2.442	−0.2
0.6	1.420	2.801	2.592	2.802	−0.04
0.7	1.606	3.168	3.024	3.178	0.3
0.8	1.798	3.546	3.456	3.569	0.6
0.9	1.994	3.933	3.888	3.969	0.9
1.0	2.192	4.323	4.320	4.378	1.2

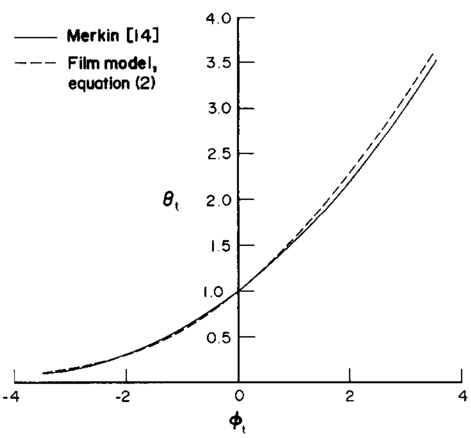

FIG. 1. The effect of transpiration on heat transfer, according to Merkin [14] and the film model.

governing equations were solved for variable physical properties. Their solution, however, differed slightly from Ostrach's [12] constant physical properties solution for air flow along an impermeable plate. Furthermore, both for suction and injection the numerical model closely agreed with experimental data. In their paper, for $Pr = 0.7$ the ratio of transpiration Nusselt number and neutral Nusselt number (denoted here by $\Theta_{t,Pari}$) was listed vs ξ (see equation (11)), and is now included in Table 3.

The dimensionless mass flux ϕ_t readily follows from equations (3)–(5) and (11), resulting in equation (12). Following Parikh *et al.* [15], the value $\psi(Pr = 0.7)$ in equation (12) reads 0.5048 (ψ is referred to in their paper as '$-f(0)$'). This value of ψ is employed here, though the trend of Table 1 indicates that Ostrach [12] obtained a somewhat smaller value: $\psi(Pr = 0.72) = 0.5046$.

In Table 3 the transformed values of ref. [15] and computed film model corrections are listed. The table illustrates that the two models agree within 5% for $-2.219 \leqslant \phi_t \leqslant 2.773$. Note that the relative difference between both models, governed by E_{Pari}, shows the same trend as the lines depicted in Fig. 1. For very negative ϕ_t the film model overpredicts heat transfer, for moderate negative ϕ_t it underestimates heat transfer rates, while for positive ϕ_t these rates are overestimated again.

Table 3. Numerical results of Parikh *et al.* [15] and the film model ($Pr = 0.7$)

Parikh *et al.* [15] ξ	$\Theta_{t,Pari}$	Equation (12) ϕ_t	Equation (2) $\Theta_{t,film}$	Equation (8) E_{Pari} (%)
2.4	0.085	−3.328	0.124	45.9
2.0	0.160	−2.773	0.185	15.6
1.6	0.260	−2.219	0.271	4.2
1.2	0.380	−1.664	0.389	2.4
1.0	0.465	−1.387	0.462	−0.6
0.8	0.550	−1.109	0.546	−0.7
0.4	0.755	−0.555	0.748	−0.9
0.0	1.000	0.000	1.000	0.0
−0.4	1.300	0.555	1.303	0.2
−0.8	1.625	1.109	1.655	1.8
−1.0	1.805	1.387	1.849	2.4
−1.2	1.990	1.664	2.053	3.2
−1.6	2.395	2.219	2.490	4.0
−2.0	2.820	2.773	2.958	4.9
−2.4	3.245	3.328	3.452	6.4

In the foregoing the attention has been focused on the effect of mass transfer on local heat transfer coefficients. In the following the effect on mean heat transfer coefficients is analysed in some detail.

The mean heat transfer coefficient is defined as

$$h_L = \psi(Pr)\frac{k}{L}\int_{x=0}^{L}\frac{1}{x}\left(\frac{Gr_x}{4}\right)^{1/4}\Theta_{t,film}\,dx \quad (13)$$

(see equations (1), (4) and (5)). For free convection with mass transfer, i.e. $\Theta_{t,film} \neq 1$, this integral, generally speaking, cannot be solved in closed form, since $\phi_{t,film}$ depends on x. Eichhorn's [13] power-law variation of the transpiration velocity ($v_w = C_2x^{-1/4}$), however, permits an analytic treatment of the integral. In this case $\phi_{t,film}$, and consequently $\Theta_{t,film}$, are constant (see equations (2)–(5)). Equation (13) then becomes

$$h_L = \psi(Pr)\frac{4k}{3L}\left(\frac{Gr_L}{4}\right)^{1/4}\Theta_{t,film}. \quad (14)$$

Hence, in this special case the mean heat transfer coefficient simply follows from multiplying the mean neutral heat transfer coefficient (which is usually documented in the literature) by the constant film model correction factor. Again it must be stressed for all other problems, e.g. uniform transpiration, that equation (13) has to be evaluated numerically.

CONCLUSIONS

In this note the film model has been applied to free convective heat transfer in the presence of mass transfer. The film model correction factor for heat transfer has been extensively compared with existing theoretical results of previous investigators. These elaborations concerned a laminar free convective boundary layer flow over an isothermal porous vertical plate. In these studies the governing equations were derived and solved numerically, the Prandtl number ranging from 0.7 to 1.

For uniform wall transpiration, the film model agreed with the literature within 5% for $-2.219 \leqslant \phi_t \leqslant 2.773$. The film model correction appeared to correlate with the same accuracy for a power-law distribution of the transpiration velocity and $-0.864 \leqslant \phi_t \leqslant 4.320$. For the said cases and ranges of ϕ_t the basic film model is well suited to describe the effect of mass transfer on free convective sensible heat transfer. Furthermore, these ranges of ϕ_t extend well beyond a large number of current practical applications.

In this note the classical film model for friction has not yet been applied to the examined physical process. For the studies referred to revealed that the exerted friction is reduced by both injection and large suction. The film model correction for friction, however, predicts enhanced and reduced friction for suction and injection, respectively (likewise heat transfer). This can be attributed to the momentum equation of the film model from which the correction factor is derived. In this equation the buoyancy and pressure gradient terms are omitted [1–4]. Though this neglect is allowed for forced convective flow, it is unacceptable for free convection. This insight may form a challenge to derive a friction correction factor from a film model with retained buoyancy and pressure gradient terms, and to apply this factor to free convective flow with transpiration.

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Transient combined conduction and radiation in an absorbing emitting and anisotropically-scattering medium with variable thermal conductivity

CHUNG-JEN TSENG and HSIN-SEN CHU

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan 30050, China

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INTRODUCTION

THE ANALYSIS of steady [1–6] or unsteady [7–10] simultaneous radiation and conduction in an absorbing, emitting and scattering medium has received extensive attention in recent years due to its wide applications in engineering. While most previous works considered constant thermal conductivity, it is well known that the thermal conductivities of most nonmetal materials are not constant when subjected to moderate, say 100 K, or large temperature differences. Instead, the thermal conductivity of most nonmetal materials displays a linear relationship with temperature and can play a significant role on overall heat transfer. In a recent study, Chu and Tseng [11] investigated the effect of variable thermal