



A new mix design concept for earth-moist concrete: A theoretical and experimental study

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ABSTRACT

This paper addresses experiments on earth-moist concrete (EMC) based on the ideas of a new mix design concept. First, a brief introduction into particle packing and relevant packing theories is given. Based on packing theories for geometric packing, a new concept for the mix design of earth-moist concrete will be introduced and discussed in detail. Within the new mix design concept, the original grading line of Andreasen and Andersen [Andreasen, A.H.M. and Andersen, J., 1930, Ueber die Beziehungen zwischen Kornabstufungen und Zwischenraum in Produkten aus losen Körnern (mit einigen Experimenten). *Kolloid-Zeitschrift* 50, p. 217–228 (in German).], modified by Funk and Dinger [Funk, J.E. and Dinger, D.R., 1994, *Predictive Process Control of Crowded Particulate Suspensions, Applied to Ceramic Manufacturing*. Kluwer Academic Press, Boston.], will be used for the mix proportioning of the concrete mixtures.

Mixes consisting of a blend of slag cement and Portland cement, gravel (4–16), granite (2–8), three types of sand (0–1, 0–2 and 0–4) and a polycarboxylic ether type superplasticizer are designed using the new mix design concept. The designed concrete mixes are tested in the lab, both in fresh and hardened states, to show the suitability of the ideas of the new mix design concept. The tested concrete mixes meet the requirements on the mechanical and durability properties.

Furthermore, the application of fine stone waste materials in the form of premixed sand (Premix 0–4) is presented. By means of an optimized particle packing, stone waste materials can be used to reduce the amount of the most cost intensive materials in earth-moist concrete mixes, viz. binder and filler. The results of tests carried out on mortar samples as well as on paving blocks produced on a laboratory paving stone machine will be discussed. The application of fine stone waste materials in earth-moist concrete mixes does not only meet the current trends in raw materials use, but also fulfill the technical requirements of the concrete in fresh and hardened state.

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1. Introduction

Earth-moist concrete mixes are the starting substance for the mass production of concrete products like pipes, slabs, paving blocks and curb stones. The working properties of earth-moist concrete (EMC), caused by its dry consistency, are advantageous. So, in comparison to ordinary vibrated concrete, the consistency of EMC allows for direct stripping of concrete products after filling and vibrating the mold. As a result, short processing times of the production process can be realized. A further example of concrete with stiff consistency is roller compacted concrete (RCC) which is stiff enough to be compacted by vibratory rollers. RCC is used for any type of industrial as well as heavy-duty pavements or in combination with bigger aggregates as roller compacted concrete for dams (RCD).

Traditional earth-moist concrete mixes for concrete products are characterized by their high cement contents between 350 and

400 kg/m³ fresh concrete and low contents of fine inert particles. They feature a low water/cement ratio ($w/c < 0.4$) combined with a very stiff consistency and a high degree of compactibility. Furthermore, the degree of consistency of EMC is defined by Häring [3] as pourable with a high degree of compaction.

The application of low w/c ratios between 0.30 and 0.35 is resulting in a low degree of hydration. This offers the possibility for post-reactions which can reduce the amount of capillary pores. The reduction in capillary pores is of vital importance for the mechanical as well as durability properties of the hardened concrete. However, compressive strength values of about 100 N/mm² as well as high values for the tensile splitting strength of 6.5 N/mm² can be achieved in the lab for cement contents of 325 kg/m³ fresh concrete. According to the national application rules of the European standard EN 1338 [4], such high strength values are not necessary for concrete products like paving blocks. This offers possibilities for further cement content optimization and improvements regarding financial and environmental aspects.

A reduction in the cement content should be possible in practice by using inert filler materials. These filler materials have to be

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implemented in the entire grading of the mix together with the binding materials in order to achieve densest possible packing. This optimized particle packing will result in a denser granular structure of the aggregates used and therefore less binding materials are needed. Owing to the denser granular structure also the mechanical properties as well as the porosity of the final product will be improved.

Different powders seems to be useful either as inert or pozzolanic filler. But besides the application of pozzolanic fillers like fly ash, the use of stone waste powders instead of inert fillers is of interest for the concrete production. During the production of washed rock aggregates, high amounts of fine stone waste powders in slurry form are generated due to the technological process. Currently, the remaining filter cake is treated as a waste material and its beneficial contribution to sustainable and environmental friendly building material is not considered. By characterizing these materials and their properties (particle size distribution, particle shape), they can replace primary raw materials like limestone or clinker.

The clinker production consumes a lot of energy and contributes in high quantities to the emission of green-house gases such as carbon dioxide. Also the production of limestone powders requires energy. Therefore, from an environmental and cost point of view, these primary raw materials should be optimally deployed for achieving mechanical and durability requirements, and the application of the appropriate industrial by-products, such as aforesaid natural stone waste, should be favored as it constitutes an environmental and financial advantage.

Moreover, these stone waste materials can contribute to further improvements regarding the workability, compactibility, green strength, and packing fraction. As an optimum packing is the key for a good and durable concrete (Brouwers and Radix [5]), the packing of all solids will be investigated at first. New ideas from the theoretical understanding of particle packing will be used for a new performance based mix design concept. This new mix design concept will be evaluated and verified by experimental tests on lab scale.

2. Particle packing in concrete mix design

The packing of solid particles is of essential importance for the understanding of granular materials used in different size classes and related problems which appear in many fields of science and industrial processes. Fig. 1 shows the size classes used for concrete ingredients and granular materials. With the fundamental understanding of the working mechanism of particle packing it is possible to control the behavior and the characteristics of products based on granular materials. Therefore, a lot of research into the field of particle packing was carried out in the last century. In particular, this research covers either the physical foundations or empirical investigations with application in industrial processes such as ceramics, chemical engineering, pharmacology and building materials.

Due to the complexity in the appearance of particles regarding their size, shape, and surface texture, particle packing applies

differently to various systems. The regular packing of equal spheres represents the simplest form of particle packing. A descriptive explanation of this phenomenon becomes more complex when the particles are packed irregularly (randomly) as different densifications are possible now.

More relevance for the packing of EMC shows the particle packing of continuous polydisperse mixtures. The packing of these polydisperse mixtures is much more complex than the packing of monosized spheres as here particles with different sizes and/or shapes are packed randomly. If the ratio of particle sizes and the ratio of pertaining quantities are constant, the packing is referred as geometric packing. Geometric packing can be subdivided in i) the packing of discretely sized particles and ii) the packing of continuously graded particle size distributions (PSDs) [7].

The packing of discretely sized systems containing of two components (bimodal mixtures) was studied by Furnas [6] and Westman and Hugill [8]. The graphical solution and their analytical expression of Westman and Hugill are mainly based on the work carried out by Furnas [6]. Later, Furnas [9] extended his work to multimodal systems and gave a solution for a continuous distribution as an extension of his solution for multimodal systems. This solution considers already a smallest and largest particle size in the mix.

Based on the properties of multimodal, discretely sized particles, De Larrard [10,11] postulated different approaches to design concrete: the Linear Packing Density Model (LPDM), Solid Suspension Model (SSM) and Compressive Packing Model (CPM). Based on the model for multimodal suspensions of Mooney [12], De Larrard [10] developed the Linear Packing Density Model composing multimodal particle mixtures. The functions of the LPDM are describing the interaction between size classes of the materials used. Due to the linear character of the LPDM, the model was improved by De Larrard [10] by introducing the concept of virtual packing density. The virtual packing density is the maximum packing density which is only attainable if the particles are placed one by one. The improvements of the LPDM resulted in the Solid Suspension Model (SSM).

In the further development of his model, De Larrard [11] introduced the compaction index to the so-called Compressive Packing Model (CPM). The compaction index considers the difference between actual packing density and virtual packing density and characterizes therefore the placing process. But also the CPM is still using the packing of monosized classes to predict the packing of the composed mixture made up of different size classes.

First attempts describing an aimed composition of concrete mixtures, which generally consists of continuously graded ingredients, can be found already more than 100 years ago. The fundamental work of Férét [13], and Fuller and Thomsen [14] showed that the packing of concrete aggregates is affecting the properties of the produced concrete. Both Férét [13] as well as Fuller and Thomsen [14] concluded that the continuous grading of the composed concrete mixture can help to improve the concrete properties. Férét [13] demonstrated that the maximum strength is attained when the porosity of the granular structure is minimal.

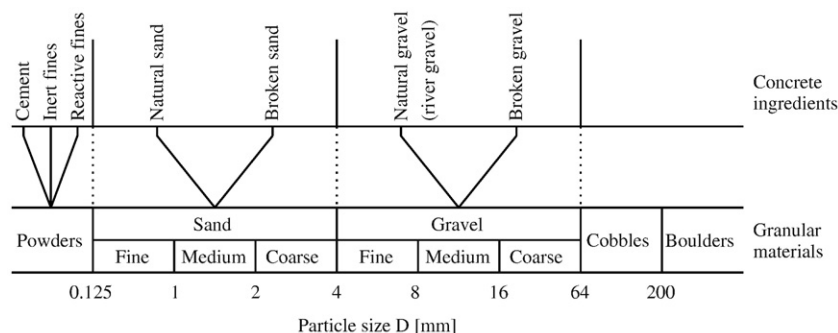


Fig. 1. Particle size ranges for granular matters and concrete aggregates.

Based on the work of Furnas [6] as well as Fuller and Thomsen [14], Andreassen and Andersen [1] studied the packing of continuously graded particles. They related their work to building materials consisting of a graded filling material (aggregates) and a binding medium as well. Based on their geometrical considerations, they proposed the following semi-empirical equation for the cumulative volume fraction:

$$P(D) = \left(\frac{D}{D_{\max}} \right)^q \quad \forall D \in [0, D_{\max}], \quad (1)$$

which considers only a maximum particle size D_{\max} in the system. Andreassen and Andersen [1] concluded from numerous experiments that the exponent q of Eq. (1) should be between 1/3 and 1/2 for densest packing. In many publications afterwards, a distribution modulus of 1/2 is referred to as 'Fuller curve' or 'Fuller parabola', based on the work of Fuller and Thomsen [14], and recommended by most design codes for conventionally vibrated concrete. Also Hummel [15] referred to the Fuller curve for composing aggregates used in standard concrete. According to Hummel [15], the grading of aggregates should follow the Fuller curve which is within the recommended area between grading curves A and B of the German standard DIN 1045-2 [16] as well as the Dutch standard NEN 5950 [17] (see Fig. 2a).

In concrete technology, another problem needs to be overcome. The grading within the recommended area between grading curves A

Table 1

Minimum amount of fine material per m³ fresh concrete [17]

Largest grain size (D_{\max}) [mm]	Minimum volume fraction fine material (<250 μm) in concrete
8	0.140
16	0.125
31.5	0.115

and B was applied to aggregates only. The limiting particle size of the recommended grading curves was defined by the DIN 1045-2 [16] and the NEN 5950 [17] for aggregates having a particle size $D > 250 \mu\text{m}$. This limitation had as a consequence that minimum amounts and/or maximum amounts of fine material are prescribed, but without considering their granulometric properties. According to the Dutch standard NEN 5950 [17], a minimum amount of fine material ($D < 250 \mu\text{m}$), depending on the maximum particle size in the composed concrete mixture, is required (Table 1). Due to requirements on the durability of the concrete on the other hand, the amount of fine material ($D < 125 \mu\text{m}$) is limited by the German standard DIN 1045-2 [16] in order to reduce the water demand of the concrete mix. Here, the maximum amount of fine particles is depending on the maximum aggregate size as well as the cement content of the composed concrete mix. Recommendations regarding the grading of fine particles ($D < 125 \mu\text{m}$ or $250 \mu\text{m}$) are not considered in the Dutch standard NEN 5950 [17] nor the German standard DIN 1045-2 [16].

As explained before, most of the design codes are setting a minimum or maximum for the amount of fine inert particles ($D < 125 \mu\text{m}$) and do not pay attention to the grading of these particles. This fact can be hindering for the optimization of the particle packing considering the entire grading of the composed mixture. But the granulometric properties of all fine materials, which are including the particle size distribution and the particle shape of the granular material, are influencing the properties of the concrete in fresh and hardened state notably [18]. A limitation in the amount of fine particles is only useful if the water content in the mix, depending on the cement content, should not exceed a certain value in order to fulfill durability properties determined by the formation of the hardening cement paste (w/c ratio). Such a restriction in the water content leads to a remarkable decrease in the workability properties of mixtures containing high amounts of fine particles and low cement contents. This decrease in the workability properties can be counteracted by the use of modern admixtures such as plasticizers.

The application of fine inert particles can help to reduce the amount of cement in the mix by using inert particles to optimize the grading of the entire mix. Due to their fineness these inert particles contribute to the green strength of the demolded product positively as they influence the capillary forces.

In Eq. (1) a minimum particle size is not considered either. Therefore, the grading is prescribed down to a particle size of zero, whereas particles smaller than $125 \mu\text{m}$ are not considered by the given grading curves. This will not be the case under practical conditions as there will be always a minimum particle size depending on the ingredients used. Accordingly, a modified version of Eq. (1) was introduced by Funk and Dinger [2] that prescribes the grading for continuously graded aggregates considering a minimum and maximum particle size in the mix. This modified equation for the PSD (cumulative volume fraction) reads:

$$P(D) = \frac{D^q - D_{\min}^q}{D_{\max}^q - D_{\min}^q} \quad \forall D \in [D_{\min}, D_{\max}], \quad (2)$$

whereby D represents the size of the sieve used for analyzing the solid ingredients. D_{\min} and D_{\max} are accounting for the minimum and maximum particle size in the mix, respectively. The distribution modulus q influences the ratio between coarse and fine particles. Higher values of the distribution modulus ($q > 0.5$) are leading to

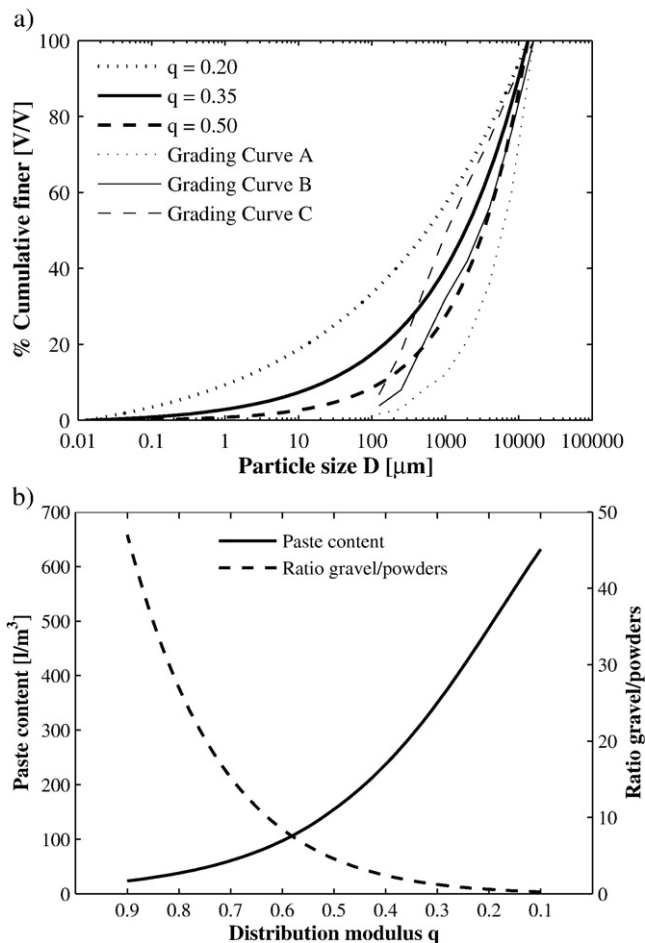


Fig. 2. a: Eq. (2) for varying distribution moduli using $D_{\max} = 16 \text{ mm}$, $D_{\min} = 0.01 \mu\text{m}$ and grading curves A, B and C according to NEN 5950 [17]. b: Influence of the distribution modulus in the modified Andreassen and Andersen equation (Eq. (2)) on the paste content per m³ fresh concrete and the ratio between gravel ($4 < D < 16 \text{ mm}$) and fines ($0.01 < D < 125 \mu\text{m}$); paste content for particles smaller than $125 \mu\text{m}$ considering a constant w/p ratio of 0.35, $D_{\max} = 16 \text{ mm}$, $D_{\min} = 0.01 \mu\text{m}$.

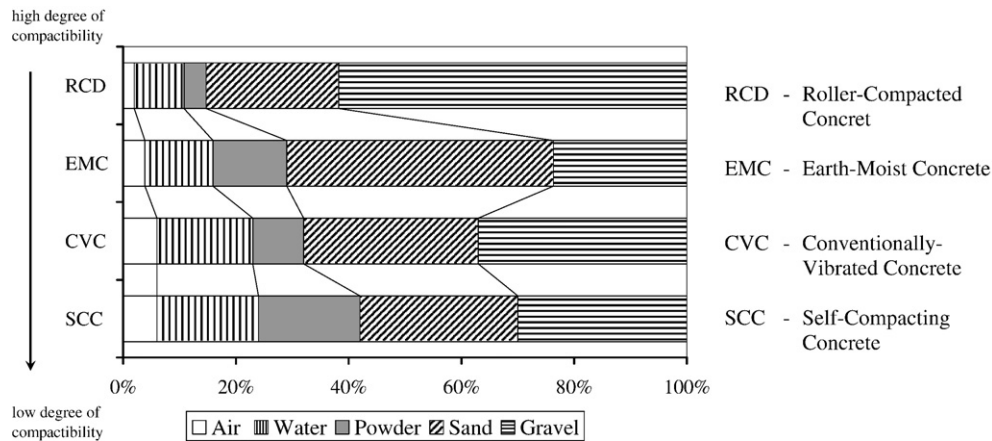


Fig. 3. Schematic composition for RCD, CVC and SCC taken from Okamura and Ouchi [24]; extended with earth-moist concrete (EMC).

coarse mixtures whereas smaller values ($q < 0.25$) are resulting in mixtures which are rich in fine particles. The influence of the distribution modulus q on the PSD of the composed mix is shown in Fig. 2a for varying q and given D_{\min} and D_{\max} . To change the ratio between coarse and fine particles in the composed mixture by changing one single parameter in the model is an important factor for the mix design. This allows the composition of ideal graded mixtures for different types of concrete having special requirements on the workability by using one equation for the aimed grading line.

Fig. 2b shows the influence of the distribution modulus q in Eq. (2) on the paste content (powders+water) and the ratio between gravel and powders. To compute the paste content as well as the ratio between gravel and powders, the volumetric amount of particles smaller than $125 \mu\text{m}$ is used. The definition 'gravel', 'powder' and other constituents, as well as the associated particle sizes, are based on the nomenclature given in Fig. 1. The different grading lines used in Fig. 2b are computed in the range between $0.01 \mu\text{m}$ (D_{\min}) and 16 mm (D_{\max}) using variant distribution moduli in steps of 0.01 and resulting in lines similar to Fig. 2a.

The computed paste content in Fig. 2b is based on constant water to powder ratio (w/p) of 0.35. The w/p ratio characterizes the amount of water in the mixtures based on the amount of powders and represents an important parameter for the workability of the composed mixture. Usually, w/p ratios of 0.35 and lower are used for EMC and stiff concrete mixtures. Higher values are applied for plastic and flowable concretes. In this consideration, the possible influence of air is ignored.

In the analyzed range of the distribution modulus, from 0.1 to 0.9, both the paste content per m^3 fresh concrete and the ratio of gravel/powders varies in a wide range. The value of the paste content per m^3 fresh concrete varies between 630 and 241 l/m^3 , while the ratio gravel/powders varies between 0.3 and 47. The extreme values in the lower and upper area of $q = 0.1$ and $q = 0.9$, respectively, are not of interest for practical applications. Brouwers and Radix [5] as well as Hunger and Brouwers [19] recommend an optimum in regard to the workability of self-compacting concrete (SCC) for $0.22 < q < 0.25$. According to suitable mix proportioning taken from literature [20–22], a range of the distribution modulus of $0.35 < q < 0.40$ is advisable for EMC. This range corresponds with own preliminary tests on EMC and meets also the recommended percentage for particles smaller than $75 \mu\text{m}$ given by Nanni et al. [23]. According to Nanni et al. [23], the percentage of aggregate passing the sieve No. 200 ($75 \mu\text{m}$) should be between 10% and 14% for RCC. This can be fulfilled using a distribution modulus of $q \geq 0.35$ and a D_{\max} of 19 mm which is typical for RCC.

As indicated by Fig. 3, taken from Okamura and Ouchi [24] and extended with EMC, the three main types of concrete require different

ratios between coarse and fine aggregates as well as dissimilar paste contents depending on their desired workability properties. According to the desired workability properties, the different types of concrete can be classified into i) flowable concretes (such as SCC) ii) plastic concretes (CVC) and iii) zero slump concretes (EMC, RCC, RCD).

In Fig. 3, EMC and RCD are standing for concretes of the same workability class but with different maximum particle sizes. The difference in the maximum particle size is necessary in order to consider the importance of the wall effect on the surface texture of the final product and the particle packing as well. Classical EMC mixtures for concrete products like paving blocks are using aggregates up to 8 mm in order to achieve a sufficient surface texture. This maximum aggregate size is higher for RCD mixes, and it can in special cases even exceed the usually used maximum aggregate size of 64 mm. However, an increase in the particle size is also resulting in a shift in the mean particle size. This shift is resulting in coarser mixtures containing less fine materials – see the difference between EMC and RCD in Fig. 3.

Furthermore, it is obvious from Fig. 3 that an increase in the paste content is related to a decrease in the degree of workability.¹ The paste content in SCC is much higher than in other types of concrete in order to achieve a flowing and stable SCC. This high content of fine particles and water is required in SCC to reduce the internal stress as the energy for flowing is consumed by the internal stress. The internal stress increases as the relative distance between bigger particles decreases and the frequency of collisions and contacts increases. The energy consumption of coarse aggregates caused by their movement relative to each other during compaction is particularly intensive. So, the internal stress can only be reduced by spreading out the coarse aggregates in SCC. The application of such high contents of fine materials in EMC hinders the achievement of densest packing associated with high green strength values as the green strength phenomenon of EMC is the result of the soil mechanical behavior in the very early age. Therefore, two opposite effects have to be considered for designing EMC mixes.

Firstly, the apparent cohesion will be influenced positively by an increasing content of fine materials as well as their fineness since the capillary forces are depending on the particle size and the grain-to-grain contacts in the finer range. Particles smaller than $125 \mu\text{m}$ have the biggest influence on the capillary forces in a granular system as with an increasing particle size the self-weight of the particles preponderate the capillary forces. Therefore, a large number of grain-to-grain contacts in the finer range are desirable.

¹ Here, workability is defined as a combination of compactability, filling behavior of the concrete mix, and demolding behavior of the fresh concrete mix after compaction.

Secondly, the inner friction will be reduced by spreading out the coarser grains by reducing their grain-to-grain contacts. Because the green strength is a result of the interaction of apparent cohesion as well as inner friction also the green strength is reduced. Furthermore, a disproportion in the ratio between coarse and fine particles in the mix will result in a grading which is not in line with the requirements given by Eq. (2) and leads to an unfavorable packing and insufficient properties of the concrete in fresh and hardened state.

Besides the packing models mentioned above, also other particle packing models have been developed. A comparison and discussion of all these models can be found in Jones et al. [25]. In the next section, a new mix design concept will be introduced considering the ideas of particle packing discussed in this section.

3. Mix design concept

In concrete, ingredients with a wide range of PSDs are combined. Therefore, different approaches can be found for the composition of concrete mixtures. The main purpose of the new mix design concept discussed here consists in the proportioning of a performance based concrete mix. This idea is realized by the formulation of an optimization problem using the modified equation of Andreasen and Andersen (Eq. (2)). The positive influence of the modified A&A equation on the properties of self-compacting concrete was already shown by Brouwers and Radix [5], and Hunger and Brouwers [19]. Furthermore, the suitability of the modified A&A equation was shown by Schmidt et al. [20], though an aimed optimization of the grading line of the composed concrete mixes in consideration of Eq. (2) was not carried out. The application of the modified A&A equation is constricted to a comparison in the shape of the curve of the various composed mixes and Eq. (2) using different distribution moduli.

Hence, it appears that an aimed composition of the concrete mix considering the grading line given by Eq. (2) can result in concrete that meet the required performance properties. Therefore, an algorithm was developed which helps to compose the concrete mix according to the PSD given by the modified A&A equation based on m ingredients ($k=1,2,\dots,m$), including the non-solid ingredients air and water. For the mix proportioning using Eq. (2), as particle size D in Eq. (2) is taken the geometric mean D_i^{i+1} of the upper and lower size of the respective

fraction obtained by sieving or laser diffraction analysis according to Eq. (3):

$$D_i^{i+1} = \sqrt{D_i D_{i+1}} \quad \text{for } i = 1, 2, \dots, n-1. \quad (3)$$

The sizes of the fractions vary in steps of $\sqrt{2}$ starting from $0.01 \mu\text{m}$ up to 125 mm (Fig. 4a). Consequently, 44 discrete sizes ($i=1,2,\dots,n+1$) are present and 43 fractions (n) are available for the classification of the $m-2$ solid ingredients.

Taking this wide range in the PSD of the granular ingredients into account, the entire grading of all aggregates, binders, and filler materials will be considered in the mix design to obtain an optimized packing. Besides the characterization of the solid ingredients regarding their PSDs, some further material properties are needed such as specific density and specific surface area. The specific density is needed as the grading line (cumulative finer fraction) of the target function is volume based, while the ingredients are dosed and analyzed (size analysis) on mass base.

The distribution modulus of Eq. (2) will be related to requirements on the workability properties and/or the paste content of the mix. As mentioned in the beginning, the mix design concept will result in the formulation of an optimization problem. The formulation of an optimization problem requires three parts which have to be defined before. These parts are:

- Target value
- Adjustable values
- Constraints.

3.1. Target value

The target value represents the objective or goal of the optimization problem. This value shall either be minimized or maximized. In the considered case, the deviation between the desired grading of the mixture and the grading given by the target function (Eq. (2)) shall be minimized. That means in particular for the mix design that the difference (residual) between the grading of the given target function $P_{\text{tar}}(D_i^{i+1})$, i.e. Eq. (2), and the grading of the composed mixture $P_{\text{mix}}(D_i^{i+1})$ shall reach a minimum value and results, in other words, in a curve fitting problem. To solve this curve fitting problem, the least squares technique

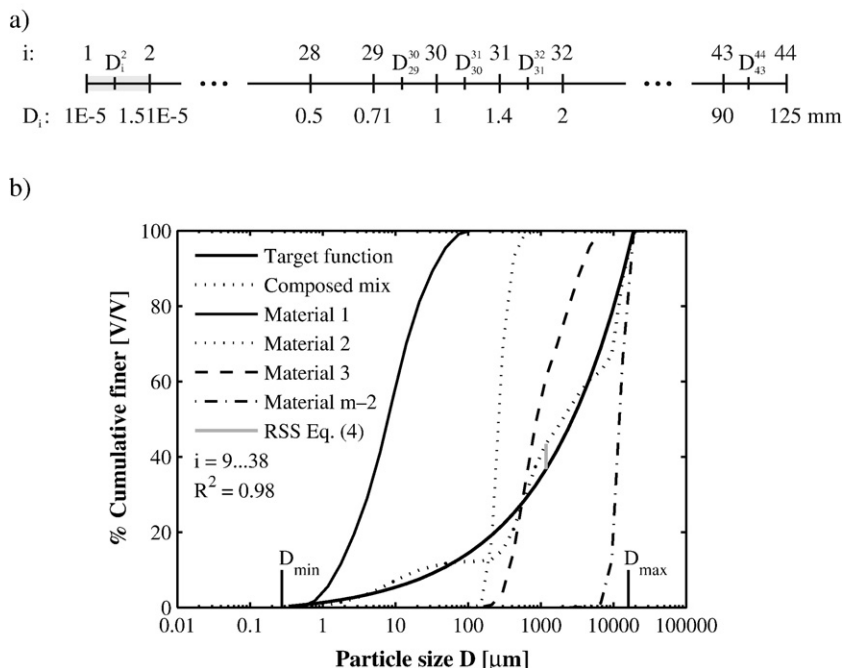


Fig. 4. a: Sizes and definition of fractions used in the optimization algorithm. b: Composed aggregate mix; $D_{\max} = 16 \text{ mm}$, $D_{\min} = 0.275$, $q = 0.35$.

is commonly used. Thereby, the sum of the squares of the residuals (RSS) is minimized. Eq. (4) expresses the least squares technique mathematically.

$$\text{RSS} := \sum_{i=1}^n e_i^2 = \sum_{i=1}^n \left(P_{\text{mix}}(D_i^{i+1}) - P_{\text{tar}}(D_i^{i+1}) \right)^2 \rightarrow \min! \quad (4)$$

$$\text{with } P_{\text{tar}}(D_i^{i+1}) = \frac{(D_i^{i+1})^q - D_{\min}^q}{D_{\max}^q - D_{\min}^q} \quad \forall D_i^{i+1} \in [D_{\min}, D_{\max}]$$

$$D_{\min} = D_i \quad \text{for } P(D_{i-1}) = 0 \wedge P(D_i) > 0$$

$$D_{\max} = D_i \quad \text{for } P(D_{i-1}) < 100 \wedge P(D_i) = 100$$

As a criterion for the evaluation of the quality of the curve fit, the coefficient of determination R^2 according to Eq. (5) is used. This value expresses the proportion of fluctuation between the target line and the obtained values for the grading of the composed mixture and is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^n (P_{\text{mix}}(D_i^{i+1}) - P_{\text{tar}}(D_i^{i+1}))^2}{\sum_{i=1}^n (P_{\text{mix}}(D_i^{i+1}) - \bar{P}_{\text{mix}})^2} \quad \forall D_i^{i+1} \in [D_{\min}, D_{\max}] \quad (5)$$

with $\bar{P}_{\text{mix}} = \frac{1}{n} \sum_{i=1}^n P_{\text{mix}}(D_i^{i+1})$ as average of the entire distribution.

3.2. Variables

The variables are adjustable values on the system. Changeable values are used by the optimization algorithm to approach the target value. Values which can be changed are the total volume of solids $V_{\text{sol}}^{\text{tot}}$ in the considered case defined as:

$$V_{\text{sol}}^{\text{tot}} = \sum_{k=1}^{m-2} V_{\text{sol},k} \quad (6)$$

and the volumetric proportion $v_{\text{sol},k}$ of each solid ingredient given by:

$$v_{\text{sol},k} = \frac{V_{\text{sol},k}}{V_{\text{sol}}^{\text{tot}}} \quad \text{for } k = 1, 2, \dots, m-2. \quad (7)$$

The volumetric proportion $v_{\text{sol},k}$ of each solid component influences the grading (computed sieve residue) of the composed mix via:

$$Q_{\text{mix}}(D_i) = \frac{\sum_{k=1}^{m-2} \frac{v_{\text{sol},k}}{\rho_{\text{sol},k}^{\text{spe}}} Q_{\text{sol},k}(D_i)}{\sum_{i=1}^n \sum_{k=1}^{m-2} \frac{v_{\text{sol},k}}{\rho_{\text{sol},k}^{\text{spe}}} Q_{\text{sol},k}(D_i)} \quad (8)$$

with

$Q_{\text{sol},k}(D_i)$ sieve residue of material k on sieve i

$\rho_{\text{sol},k}^{\text{spe}}$ specific density of material k .

The computed cumulative finer fraction of the composed mix is given by:

$$P_{\text{mix}}(D_i^{i+1}) = \begin{cases} P_{\text{mix}}(D_{i-1}^i) - Q_{\text{mix}}(D_i) & \text{for } i = 1, 2, \dots, n-1 \\ 1 & \text{for } i = n \end{cases} \quad (9)$$

As mentioned before, the total volume of solids per m^3 fresh concrete is also changed by the optimization algorithm. The volumetric amount of solids per m^3 fresh concrete is not directly connected with the target value. Here, a connection exists via the constraints.

3.3. Constraints

Constraints are restrictions connected with the adjustable values and/or the target value and reflect real-world limits or boundary

conditions. These restrictions are expressed in the form of a system of equations or inequations. The formulation of an optimization problem distinguishes between physical constraints and policy constraints.

Physical constraints are determined by the physical nature of the optimization problem. For the formulated optimization problem the following physical constraints result from the physical boundary conditions.

3.3.1. Non-negativity constraint

This constraint considers that a negative volumetric proportion $v_{\text{sol},k}$ of each solid component in Eq. (8) as well as a negative total volumetric amount of solids $V_{\text{sol}}^{\text{tot}}$ per m^3 fresh concrete are not an admissible solution:

$$v_{\text{sol},k} > 0 \quad \text{for } k = 1, 2, \dots, m \quad (10)$$

3.3.2. Volumetric constraint

The volumetric constraint takes into account that the sum of the volumetric proportion $\sum v_{\text{sol},k}$ of the granular ingredients used in Eq. (8) cannot be higher than 1. Moreover, the total volume of all ingredients (including air and water) per m^3 fresh concrete, according to Eq. (12), cannot be higher or lower than 1 m^3 :

$$\sum_{k=1}^{m-2} v_{\text{sol},k} = 1 \quad (11)$$

$$V_{\text{con}} = V_{\text{sol}}^{\text{tot}} + V_{\text{wat}} + V_{\text{adm}} + V_{\text{air}} = V_{\text{agg}} + V_{\text{cem}} + V_{\text{fil}} + V_{\text{wat}} + V_{\text{adm}} + V_{\text{air}} = 1 \text{ m}^3 \quad (12)$$

Eq. (12) expresses the volumetric connection of the solid ingredients (aggregates V_{agg} , binders (cement) V_{cem} , fillers V_{fil} , and admixtures V_{adm}) as well as the water V_{wat} , and the air content V_{air} . The air content per m^3 fresh concrete is estimated a priori to be 0.04 m^3 ($=4.0\%$), this value needs to be verified later as it is depending on the maximum packing fraction of all solids, on the water content and on the applied compaction efforts. Furthermore, Eq. (12) contains volumetric contents which can be described through further relations between the solid ingredients.

One of the most characteristic values is the water to cement ratio (w/c) used. The amount of water in the mix against the cement content defines the w/c ratio:

$$w/c = \frac{M_{\text{wat}}}{M_{\text{cem}}} = \frac{\rho_{\text{wat}} V_{\text{wat}}}{\rho_{\text{cem}}^{\text{spe}} V_{\text{cem}}} \quad (13)$$

The applied w/c ratio of the mixture is influencing the formation of the hardened cement paste as well as its microstructure. Besides the major influence of an optimized packing, the microstructure of the hardened cement paste affects the strength development of the concrete mix.

Regarding the workability of the designed concrete mixture, the water to powder ratio w/p is of greater interest for concrete mixtures having low cement contents:

$$w/p = \frac{M_{\text{wat}}}{\sum_{k=1}^{m-2} M_{\text{sol},k}} = \frac{\rho_{\text{wat}} V_{\text{wat}}}{\sum_{k=1}^m \rho_{\text{sol},k}^{\text{spe}} V_{\text{sol},k}(D_i)} \quad \text{for } D_i < 125 \text{ } \mu\text{m} \quad (14)$$

In this case, the w/p ratio is used for all particles in the mixture with a particle size smaller than $125 \text{ } \mu\text{m}$. They may originate from cement, filler and aggregate (e.g. fine sand). Both the w/c ratio and the w/p ratio can be chosen as a constraint for the optimization. By choosing the w/c ratio or the w/p ratio, the amount of water in the mix will be determined. Consequently, both the w/c and the w/p ratio are connected with the volumetric constraint. But the w/c ratio itself is considered as a policy constraint, whereas the w/p ratio represents a logical constraint.

Table 2
Properties of materials used

Material	Type	$\rho^{\text{spe}} [\text{g}/\text{cm}^3]$
Cement	CEM I 52.5 N	3.064
Cement	CEM III/B 42.5 N LH/HS	2.962
Fine sand	Sand 0–1 mm	2.636
Medium sand	Rhine sand 0–2 mm	2.650
Coarse sand	Rhine sand 0–4 mm	2.642
Premixed sand	Premix 0–4 mm	2.648
Broken granite	Granite 2–8 mm	2.650
Gravel	Rhine gravel 2–8 mm	2.620
Gravel	Rhine gravel 4–16 mm	2.605
Superplasticizer	Glenium 51	1.100

3.3.3. Policy and logical constraints

Policy and/or logical constraints represent requirements given by standards (policy constraints) or particular requirements (logical constraints) on the designed concrete. In general, these requirements can be of different nature. These constraints are also linked via the variables with the target value. The remaining constraints in the optimization problem are as follows:

- Minimum cement content $V_{\text{cem}}^{\text{min}}$
- Maximum cement content $V_{\text{cem}}^{\text{max}}$
- w/c ratio or w/p ratio
- ratio between binder 1 ($V_{\text{cem},1}$) and binder 2 ($V_{\text{cem},2}$) – if two different binders are used
- ratio between Portland cement and slag – if a slag cement with individual composition shall be considered.

The optimization problem can be formulated and solved numerically when the target function, material properties, variables, and all constraints are specified. The volumetric content of aggregates, binder, filler material, and water is given by the solution of the optimization problem. The solution of the optimization problem leads to a PSD of the solid mix which follows the given target function with a minimum deviation. The mix proportioning in terms of mass follows by accounting for the specific densities of the ingredients.

Fig. 4b shows as an example the grading of a composed aggregate mix based on 4 solid ingredients ($m=6$). Furthermore, the applied target function ($P_{\text{tar}}(D_i^{j+1})$) as well as the PSDs of the materials used and the resulting maximum (D_{max}) and minimum particle size (D_{min}) are also depicted in Fig. 4b. This figure illustrates that by combining four ingredients only, that each have a PSD that significantly differs from Eq. (2), a mix can be designed that closely follows the given target PSD (Eq. (2)).

4. Concrete experiments

To demonstrate the suitability of geometric packing for earth-moist concrete and the application of the modified A&A equation, several concrete mixes have been designed and tested in the lab. The

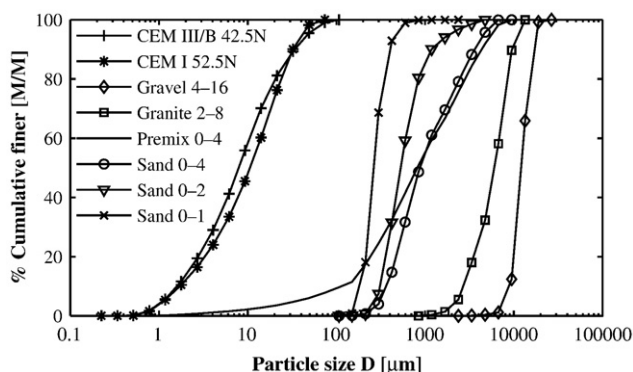


Fig. 5. PSDs of aggregates and powders used (cumulative finer mass fraction).

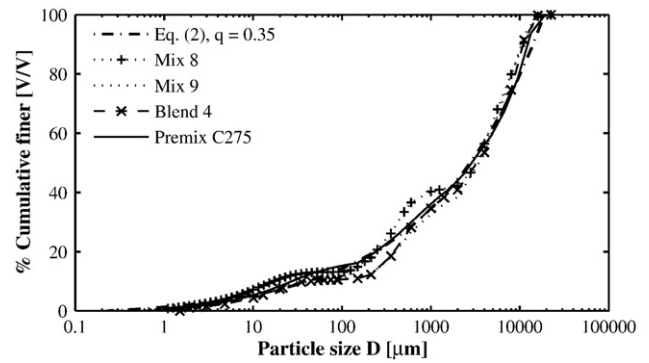


Fig. 6. PSDs of selected earth-moist concrete mixtures tested in the lab (cumulative finer volume fraction).

designed EMC mixtures are solely based on cement and aggregates without using additional fillers. Besides the tests carried out in the lab, some additional tests are performed on paving blocks produced on a laboratory paving block machine. The results of these tests will also be discussed in detail in this section.

By means of the new mix design concept and the optimization tool discussed in the previous section, EMC mixes using different types of material are designed. Table 2 gives an overview over the material properties of the solid ingredients used for the concrete mix design and the lab tests. The PSDs of the applied solid ingredients are given in Fig. 5. The PSDs of the sands and gravels is determined by sieving, the PSDs of cement and fine materials ($D < 125 \mu\text{m}$) by laser granulometry. Materials that contain both gravel and/or sand, and powder smaller than $125 \mu\text{m}$ are separated at $125 \mu\text{m}$ and analyzed separately using sieve analysis (gravel and sand fractions) as well as laser granulometry (powder fraction).

In Fig. 6 the grading of some composed concrete mixes is shown. The mix proportioning of the tested mixes is given in Table 3. The designed concrete mixes are tested both in fresh and hardened state.

The consistence of the concrete mix in fresh state is assessed by the degree of compactibility c_{DIN} according to DIN-EN 12350-4 [26].

Table 3
Mix proportioning of composed concrete mixes

Mix	CEM I 52.5 N	CEM III/B 42.5 N LH/HS	Sand				Gravel				Water	SP
	[kg]	[kg]	0–1	0–2	0–4		2–8	4–16	8–16			
			[kg]	[kg]	[kg]		[kg]	[kg]	[kg]		[kg]	[kg]
Mix 1	–	310.0	482.8	–	475.4	584.8	366.0	–	–	139.5	–	
Mix 2	–	310.0	482.8	–	475.4	584.8	366.0	–	–	139.5	–	
Mix 3	–	310.0	88.6	–	594.1	818.7	448.8	–	–	124.0	–	
Mix 4	–	310.0	227.8	–	604.7	605.0	512.7	–	–	124.0	–	
Mix 5	–	310.0	92.0	–	599.5	826.2	452.9	–	–	116.2	–	
Mix 6	–	310.0	92.0	–	599.5	826.2	452.9	–	–	116.2	0.63	
Mix 7	–	310.0	92.0	–	599.5	826.2	452.9	–	–	116.2	0.94	
Mix 8	–	310.0	92.0	–	599.5	826.2	452.9	–	–	116.2	0.93	
Mix 9	–	290.0	120.1	–	655.7	628.1	585.3	–	–	116.0	–	
Mix 10	–	239.5	2.3	–	715.0	773.3	613.4	–	–	89.8	–	
Mix 11	–	310.0	400.2	–	522.7	599.2	448.6	–	–	116.3	0.63	
Mix 12	–	310.0	400.2	–	522.7	599.2	448.6	–	–	116.3	0.94	
Blend 1	130.0	245.0	–	698.0	–	–	–	–	356.0	131.3	–	
Blend 2	112.7	212.3	–	602.0	–	–	–	–	396.9	113.7	–	
Blend 3	112.7	212.3	–	602.0	–	–	–	–	396.9	113.7	1.63	
Blend 4	112.7	212.3	–	602.0	–	–	–	–	396.9	113.7	0.98	

SP: superplasticizer.

The degree of compactability is computed using the distance s_i between the top edge of a square container sizes $a \times a \times h_1$ of $200 \times 200 \times 400 \text{ mm}^3$ and the surface of the concrete after compaction in the middle of each side of the container. The equation for the degree of compactability c_{DIN} according to DIN-EN 12350-4 [26] is as follows:

$$c_{\text{DIN}} = \frac{h_1}{h_1 - \bar{s}} = \frac{h_1/\bar{s}}{h_1/\bar{s} - 1}; \quad \bar{s} = \frac{1}{4} \sum_{i=1}^4 s_i. \quad (15)$$

Due to a simplification of the test procedure, the determination of the degree of compactability took place by using the molds used for the compressive strength test. By means of this procedure, the results of the compressive strength test and the degree of compactability can be directly related as they are determined on the same sample. For this purpose, standard cubes with side length a of $150 \times 150 \times 150 \text{ mm}^3$ are employed with a supplementary upper part of 150 mm height. So, the total height h_1 of the rectangular cast is 300 mm instead of 400 mm given in DIN-EN 12350-4 [26] and shown in Fig. 7.

It is assumed that the downsizing of the container will not influence the results as a constant scaling factor of 0.75 is used for all dimensions of the container so that theoretically the resulting c_{DIN} will be the same (see Eq. (15)). This fact is also confirmed by the linear relation between the degree of compactability c_{DIN} according to DIN-EN 12350-4 [26] and the computed degree of compactability c_{rho} shown in Fig. 8. The computed degree of compactability c_{rho} uses the densities of the fresh concrete densely and loosely packed in a round vessel having a fixed volume of 8 l and a diameter of 205 mm. Considering the determined densities of the fresh concrete mixes, the equation for the computed degree of compactability c_{rho} is as follows:

$$c_{\text{rho}} = \frac{\rho_{\text{con}}^{\text{den}}}{\rho_{\text{con}}^{\text{loo}}} = \frac{M_{\text{con}}^{\text{den}}/V_{\text{ves}}}{M_{\text{con}}^{\text{loo}}/V_{\text{ves}}}. \quad (16)$$

Furthermore, the packing fractions of the fresh concrete mixes in loose as well as dense state can be computed using the measured values of the density test as follows:

$$\text{PF} = \frac{V_{\text{sol}}}{V_{\text{ves}}} = \frac{\sum_{k=1}^{m-2} V_{\text{sol},k}}{V_{\text{ves}}} = \frac{\sum_{k=1}^{m-2} \frac{M_{\text{sol},k}}{\rho_{\text{sol},k}}}{V_{\text{ves}}}. \quad (17)$$

For determining the packing fraction, the vessel of the air entrainment meter was filled with the fresh concrete and weighted before and after compaction using a constant compaction effort. The weight of the concrete sample inside the vessel is used for computing the packing fraction considering the volumetric mix proportioning of

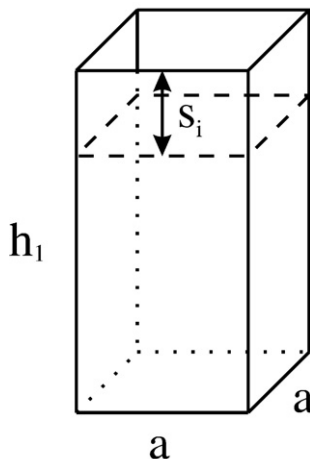


Fig. 7. Sizes of the square container described in DIN-EN 12350-4 [26].

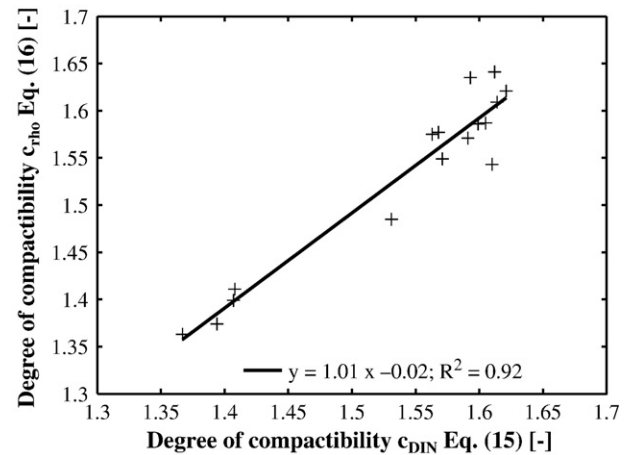


Fig. 8. Relation between the degree of compactability c_{DIN} according to DIN-EN 12350-4 [26] and the computed degree of compactability c_{rho} using density values.

the designed concrete mix as well as the specific densities of the concrete ingredients. Furthermore, the packing fraction can be used to calculate the void fraction of the mixtures as follows:

$$\phi = \frac{V_{\text{wat}} + V_{\text{air}}}{V_{\text{ves}}} = \frac{V_{\text{ves}} - V_{\text{sol}}}{V_{\text{ves}}} = 1 - \text{PF}. \quad (18)$$

The obtained results for the degree of compactability and the packing fraction are depicted in Fig. 9.

The tested EMC mixtures gave values between 1.15 and 1.65 for the degree of compactability. These values are in the range for which this measuring method is applicable. According to Bonzel and Krell [27], this test method is suitable for concrete mixtures obtaining values higher than 1.10 for the degree of compactability. EMC mixtures show a high degree of compactability of 1.5 and higher, whereas stiff concrete mixtures are in the range between 1.45 and 1.25. Plastic concrete mixes result in values between 1.25 and 1.05 and SCC mixes have a degree of compactability of around 1, as they are nearly not compactable.

Moreover, it is obvious from Fig. 9 that concrete mixtures having a high degree of compactability are showing higher packing fractions (in fresh state). Accepting a constant loose packing fraction for the tested concrete mixtures, this was expected as the degree of compactability governs the volume difference of a defined concrete mass before and after compaction. Therefore, high values for the degree of compactability are caused by a high volume difference within the system and a denser granular structure after compaction. Additionally, a high value

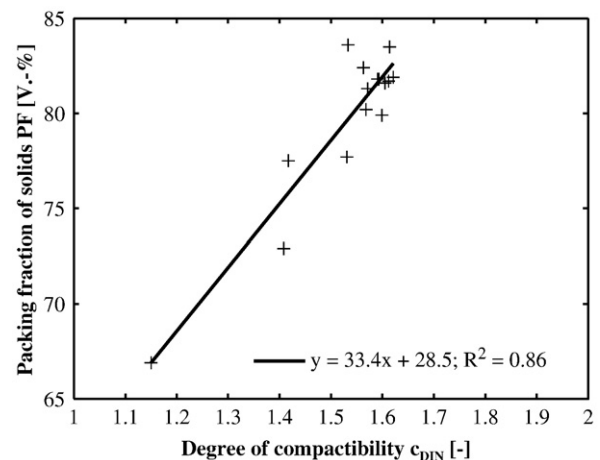


Fig. 9. Measured packing fraction versus degree of compactability for tested EMC mixes.

of the degree of compactibility indicates a better compactibility of the concrete mixture as the volume change after compaction (assuming an identical compaction effort) is also higher.

The designed mixes have been poured in standard cubes, cured sealed during the first day, demolded, and subsequently cured for 27 days submersed in a water basin at 20 ± 2 °C. After 28 days, the cubes are tested for compressive strength and tensile splitting strength. The compressive strength f_c of each cube is determined according to the standard DIN-EN 12390-4 [28]. The mean values of the compressive strength of each tested series, based on three cubes per series, are given in Table 4 and depicted in Fig. 10a.

The obtained values presented in Fig. 10a show a linear relation between the fresh packing fraction and the compressive strength of the hardened concrete. This holds for both mixtures using only slag cement (CEM III/B 42.5 N LH/HS) and mixtures containing a blend of 65% slag cement and 35% Portland cement (CEM I 52.5 N). The amount of cement in the different mixtures using only slag cement varies between 290 and 320 kg/m³ fresh concrete. The amount of cement in mixtures using the blend of both cements was fixed to 325 kg/m³ fresh concrete. The linear relation between the packing fraction and the compressive strength is an important fact for reducing the amount of cement to a minimum amount necessary for fulfilling the mechanical requirements. As the amount of cement per m³ fresh concrete as well as the applied w/c ratio is varying marginally, it can be assumed that the increase in the compressive strength for mixes having a higher packing fraction is caused by an improved granular structure. The positive relation between packing fraction and mechanical properties is one of the basic features of the new mix design concept.

Another important conclusion in regard to cement efficiency is also obvious from Fig. 10a. Not only the compressive strength of a mixture can be improved by an optimized and dense packing of all granular ingredients, but also the cement can be used more efficiently. This is indicated by the compressive and flexural strength cement efficiency x_c of the concrete mixture and described by:

$$x_c = \frac{f_c}{M_{cem}} \quad \text{and} \quad x_f = \frac{f_t}{M_{cem}}, \quad (19)$$

respectively. Based on the values given in Fig. 10a the cement efficiency regarding the compressive strength could be increased for mixtures using only slag cement, from 0.13 up to 0.21 N m³/kg mm². For mixtures using a blend of slag cement and Portland cement the cement efficiency could even be improved from 0.22 to 0.31 N m³/kg mm² by means of an optimized particle packing.

Table 4
Mechanical properties and obtained packing fractions for tested concrete mixtures

Specimen	Compressive strength f_c [N/mm ²]	Tensile splitting strength f_{ct} [N/mm ²]	Packing fraction PF [V.-%]	Cement efficiency x_c [N m ³ /kg mm ²]
Mix 1	41.1	–	77.5	0.133
Mix 2	36.9	–	77.7	0.119
Mix 3	48.8	–	81.9	0.157
Mix 4	52.3	–	81.8	0.169
Mix 5	48.2	–	81.7	0.156
Mix 6	47.3	–	79.9	0.153
Mix 7	49.9	–	81.8	0.161
Mix 8	63.7	–	83.6	0.205
Mix 9	51.6	–	82.4	0.178
Mix 10	14.1	–	72.9	0.059
Mix 11	43.0	–	80.2	0.139
Mix 12	48.0	–	81.3	0.155
Blend 1	82.6	4.92	81.6	0.216
Blend 2	83.5	4.97	83.5	0.235
Blend 3	95.3	–	84.0	0.279
Blend 4	100.2	–	84.7	0.308
Premix C275	23.6	1.79	71.6	0.086
Premix C275	48.6	3.81	81.6	0.176

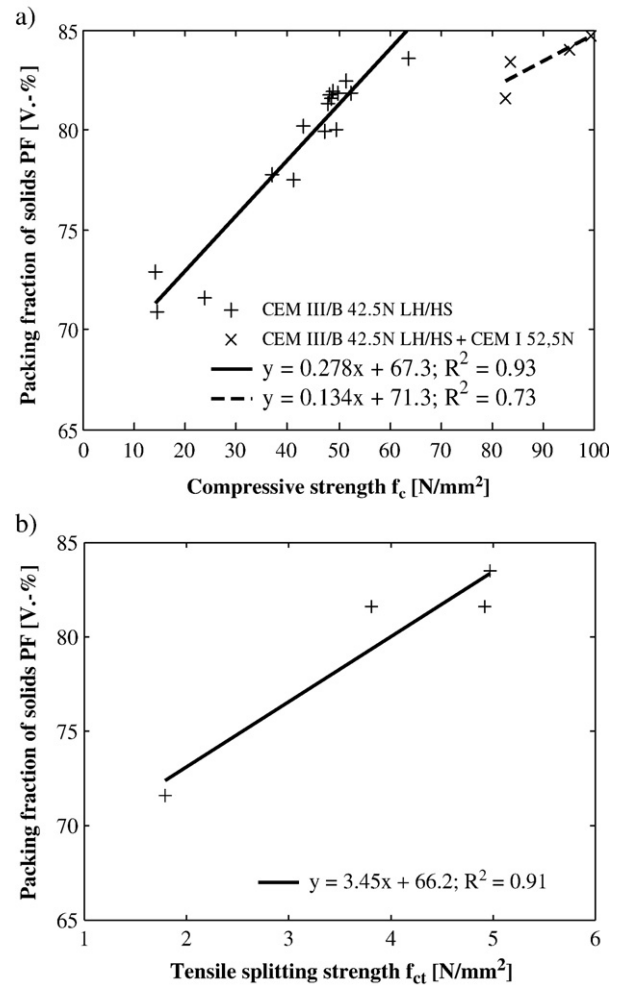


Fig. 10. a: Measured packing fraction versus compressive strength. b: Measured packing fraction versus tensile splitting strength.

Next, the tensile strength is measured for some selected mixtures using the indirect tensile splitting test. For testing the tensile splitting strength of the cubes, the conditions given by the standard DIN-EN 12390-6 [29] are applied. The tensile strength follows from:

$$f_{ct} = \frac{F}{\pi a^2}. \quad (20)$$

The resulting mean values are listed in Table 4 and depicted in Fig. 10b. Although a few samples have been analyzed, the results obtained for the tensile splitting strength are also depending on the packing fraction and seem to increase also with an increasing packing fraction.

5. Application of stone waste materials

During the production of washed rock aggregates, high amounts of fine stone waste powders in slurry form are generated throughout the washing process. Also the production of ornamental natural stone slabs generates high amounts of fines as a by-product of sawing, polishing, etc. Currently, the remaining filter cake is treated as a waste material and its beneficial contribution to sustainable and environmental friendly building materials is not considered.

Depending on the origin and the generation, two different options are possible for the application of the fine stone waste materials from rock production in concrete. The first possibility is based on the use of the generated filter cake, for instance as a redispersion of the remaining filter cake in slurry form. This approach is suitable if the material is

Table 5a

Mix proportioning of composed concrete mixes containing stone waste material (Premix 0–4)

Mix	CEM III/B 42.5 N LH/HS [kg/m ³]	CEM I 52.5 N [kg/m ³]	Premix 0–4 [kg/m ³]	Granite 2–8 [kg/m ³]	Gravel 8–16 [kg/m ³]	Water [kg/m ³]	w/c	w/p
Premix C275	179.7	95.3	999.6	377.4	583.3	118.3	0.43	0.33
Premix C250	175.0	75.0	1034.5	346.8	591.8	121.6	0.49	0.33
Premix C200	140.0	60.0	1140.3	283.6	626.6	109.1	0.55	0.33
Premix C175	122.5	52.5	1182.9	252.0	638.8	108.7	0.62	0.35

already generated or the generation of fine stone waste materials cannot be avoided (e.g. through cutting and polishing processes).

The second possibility considers the direct use of the untreated or unwashed product. In this case, the stone waste material will not be generated as the original product allows a direct use of the material in special types of concrete. This method will result in a higher financial and environment-friendly aspect as an intermediate step in the production of broken rock aggregates is eliminated. Therefore, the direct use of this untreated product, its sand fraction here named Premix 0–4, is of major interest.

By characterizing these materials and their properties (particle size distribution, particle shape), Premix 0–4 can replace primary raw materials like limestone powder or clinker. This results in a second benefit, as the clinker production consumes a lot of energy and contributes in high quantities to the emission of green-house gases such as carbon dioxide. Also the production of limestone powders requires energy. Therefore, from an environmental and cost point of view, these primary raw materials should be optimally deployed for achieving the mechanical and durability requirements, and the application of the appropriate industrial by-products should be favored. As discussed in the previous section, it was shown that packing plays an important role. Using the new mix design tool, mixes can be developed in which cement is partly replaced by the fines of the Premix 0–4 (see Fig. 5).

Based on the results of the material characterization, particularly with respect to particle size distribution at present, four different EMC mixes are designed by means of the ideas of the new mix design concept. The designed mixes are using premixed sand (Premix 0–4), containing both fine aggregate and inert stone powder, in combination with varying cement contents. The distribution modulus q is chosen to be 0.35 for all mixes. Considering a distribution modulus of $q=0.35$ and a w/p ratio of 0.35, the necessary cement content amounts to 235 kg/m³ fresh concrete to follow the given target line with the lowest deviation.

For the investigations, the cement content is reduced starting from 275 kg down to 175 kg/m³ fresh concrete, this being compensated by higher amounts of Premix 0–4 and gravel as well. For designing the

Table 5b

Mix proportioning of tested mortar mixes derived from Table 5a

Mix	CEM III/B 42.5 N LH/HS [kg/m ³]	CEM I 52.5 N [kg/m ³]	Premix 0–4 [kg/m ³]	Granite 2–4 [kg/m ³]	Water [kg/m ³]	Plasticizer [kg/m ³]	w/c	w/p
Premix C275	262.8	139.5	1462.3	178.9	173.0	2.00	0.43	0.33
Premix C250	254.3	109.0	1503.2	163.3	176.7	2.05	0.49	0.33
Premix C200	202.5	86.8	1649.7	133.0	157.9	1.86	0.55	0.33
Premix C175	176.4	75.6	1702.9	117.5	156.5	1.75	0.62	0.35

Table 6a

Mean compressive strength of tested mortar samples

Mix	Compressive strength f_c [N/mm ²]			Cement efficiency α_c [N m ³ /kg mm ²]
	3 days	7 days	28 days	
Premix C275	–	44.9	61.7	0.224
Premix C250	28.6	36.8	63.9	0.255
Premix C200	13.8	25.2	38.5	0.192
Premix C175	16.0	34.6	52.0	0.297

concrete mixtures, a blend of 65% slag cement (CEM III/B 42.5 N LH/HS and 35% Portland cement (CEM I 52.5 N) was used. Owing to the high content of fines and low cement contents in the mixtures, the amount of water is maintained as low as possible in order to achieve w/c ratios around 0.50. There, the use of plasticizers is necessary to match the industrial compaction efforts on the packing with available compaction efforts under laboratory conditions. The detailed mix proportioning of the designed concrete mixes is given in Table 5a.

The designed EMC mixes are tested on mortar scale as mortars permit a quick and handy test method of preliminary mix designs. For conducting the tests on mortar scale, all ingredients smaller than 4 mm of the designed concrete mixes given in Table 5a are used. The resulting mix proportion of the mortar mixes is given in Table 5b.

The mortar samples are tested regarding their compressive strength, flexural strength as well as their water absorption. For testing the compressive strength, cubes with dimensions of 50×50×50 mm³ have been produced using constant compaction efforts and tested after 3, 7 and 28 days. The mean values of the compressive strength tests are given in Table 6a and depicted in Fig. 11a.

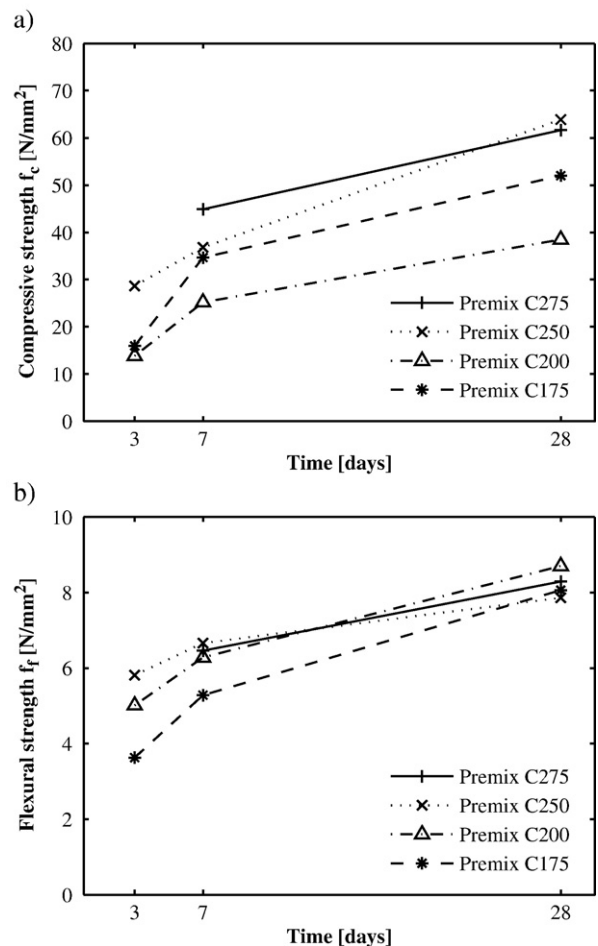


Fig. 11. a: Mean compressive strength of tested mortar samples. b: Mean flexural strength of tested mortar prisms.

The compressive strength after 28 days of the tested samples having a cement content of 275 and 250 kg/m³ fresh concrete is not influenced by the cement content of the designed mixtures. Here, the mix having a cement content of 250 kg cement/m³ fresh concrete achieves the same compressive strength after 28 days as the mix having 275 kg. The optimum cement content for following the given target line with the lowest deviation amounts to 235 kg. This amount of cement results from the properties of the used materials and using a distribution modulus of $q=0.35$ in Eq. (2).

It appears that a reduction in the cement content is not influencing the compressive strength when the original cement content is already higher than actually needed. In this case the additional cement only acts as a kind of filling material. The reduction in the cement content shows higher effect on the compressive strength if the cement content is already below the necessary amount needed for optimum packing, and a further reduction in the cement content is then influencing the granular structure in a negative way.

Considering the data of the mix proportioning given in Table 5b, the w/p ratio and the workability is constant for mixes having a cement content of 275, 250 and 200 kg/m³ fresh concrete. But the w/p ratio was increased from 0.33 to 0.35 for the mix having 175 kg/m³ fresh concrete. This slight increase in the water content improved the workability properties of the mixture and the granular structure of the hardened concrete. Therefore, the mix containing 175 kg cement/m³ fresh concrete achieved higher compressive strength values than the mix using 200 kg cement/m³ fresh concrete and is resulting in the highest cement efficiency of all mixes. The highest cement efficiency, considering a constant w/p ratio of 0.33, was achieved for the EMC mix using 250 kg cement/m³ fresh concrete. The present results show therefore that cement can be used more efficient when it results in an optimized particle packing.

The development of the flexural strength of the mortar samples was also determined after 3, 7 and 28 days on prisms with dimensions of 40×40×160 mm³. The mean values of each test series are given in Table 6b and presented in Fig. 11b.

Significant variations in the flexural strength in dependence on the varying cement contents are only recognizable for the strength development up to 7 days. The effect of cement reduction clearly on the results of the flexural strength tests after 28 days is hardly visible as the standard deviation for each particular series is higher than the difference of the mean values among each other. All tested series using Premix 0–4 showed high flexural strength values in the range between 7.9 and 8.1 N/mm². Due to the marginal difference between the values of the flexural strength, the cement efficiency regarding flexural strength is increasing with decreasing cement contents. This shows clearly that the cement can also be used in a more efficient way when the flexural strength is considered.

Based on the results of the mortar experiments, one of the mixes containing Premix 0–4 was selected for further tests using a laboratory paving block machine. The produced paving stones in single-layer technique are having dimensions of 198×198×80 mm³. For the production of the paving blocks, mix Premix C250 from Table 5a was chosen and now produced including the coarse aggregate fractions according to the mix proportioning given in Table 5a.

The produced paving blocks have been tested regarding their tensile splitting strength and water absorption according to the European

Table 7

Experimental results of the tested paving blocks and limiting values of the European standard EN 1338 [4]

	Tensile splitting strength T [N/mm ²]	Breaking load L_{spl} [N/mm]	Water absorption (M.-%)	Density oven dry [g/cm ³]	Open porosity [V.-%]
Average	6.4	839	3.0	2.48	7.5
Lowest single value	6.4	832	–	–	–
Standard deviation	0.06	6.77	–	–	–
Coefficient of variation [%]	0.9	0.8	–	–	–
Limit EN 1338	Characteristic strength >3.6; lowest single value >2.9	>250	Not required for class A; avg. ≤6 for class B	Not required	Not required

standard EN 1338 [4]. The tensile splitting strength of paving blocks is not calculated with Eq. (20) but follows, depending on the thickness of the paving block t , from:

$$T = 0.637 \times k \times \frac{F}{a_{spl} t} \quad (21)$$

$$\text{with } k = \begin{cases} 1.3 - 30(0.18 - t/1000)^2 & \text{for } 40 \text{ mm} \leq t \leq 180 \text{ mm} \\ 1.3 & \text{for } t > 180 \text{ mm} \end{cases}$$

In addition to the tensile splitting strength, the breaking load of the paving block has to be related to the length of the splitting area, the so-called length-related breaking load, via:

$$L_{spl} = \frac{F}{a_{spl}} \quad (22)$$

Furthermore, the open porosity of the stones has been determined. The obtained values are also given in Table 7 in comparison with the limiting values of the European standard EN 1338 [4]. The time dependent behavior of the water absorption is given in Fig. 12.

The tested paving blocks fulfill the requirements on the flexural strength given by the European standard EN 1338 [4] as well as length-related breaking load. The achieved values for the tensile splitting strength are about two times higher than the required limit of 3.6 N/mm². In the same way, the length-related breaking load exceeds the required limit by a factor of three. That offers the possibility for further reduction in the cement content of the mix.

Furthermore, the water absorption of the tested blocks is also in line with the requirements given by the EN 1338 [4]. The time dependent behavior of the water absorption is depicted in Fig. 12. The

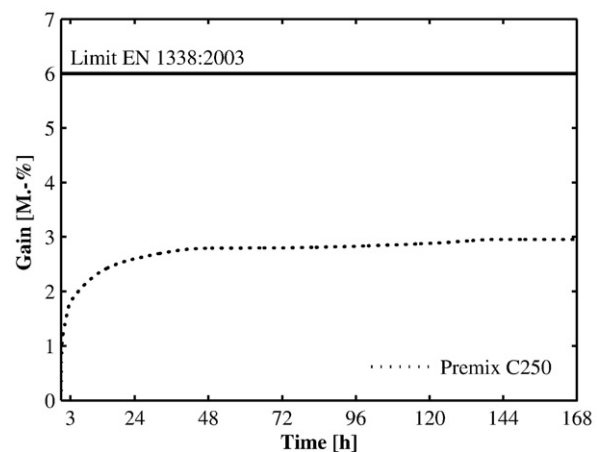


Fig. 12. Water absorption of paving blocks produced on a laboratory paving block machine.

Table 6b

Mean flexural strength of tested mortar prisms

Mix	Flexural strength f_f [N/mm ²]			Cement efficiency η_f [N m ³ /kg mm ²]
	3 days	7 days	28 days	
Premix C275	–	6.5	8.3	3.02E–02
Premix C250	5.8	6.7	7.9	3.16E–02
Premix C200	5.0	6.3	8.7	4.35E–02
Premix C175	3.6	5.3	8.1	4.63E–02

results of the water absorption test yield that the mix can be classified as durable since the water absorption is an indicator for the durability of the tested concrete blocks.

6. Discussion

The tests on EMC showed that a general relation between the distribution modulus of the modified A&A equation and the packing fraction can be derived. Based on the data obtained from the concrete experiments, the relation between distribution modulus and packing fraction is shown in Fig. 13. It becomes clear from Fig. 13 that highest packing fractions have been achieved for a distribution modulus of $q=0.35$. The same fact applies for the compressive strength of the hardened concrete. Here, mixes with a distribution modulus of $q=0.35$ and high packing fractions achieved higher compressive strength values than the mixes with lower packing fractions, caused by their lower distribution moduli.

Besides the use of suitable distribution moduli for composing concrete aggregates, some further properties of the fresh concrete mix are also important for workable concrete mixes and partly influenced by the chosen distribution modulus. Based on the analysis of the obtained test results, the following values are advisable for the characterization of workable EMC mixes:

- Distribution modulus (q): 0.35–0.40
- Paste content ($<125\ \mu\text{m}$): $0.225\text{--}0.250\ \text{m}^3$ per m^3 fresh concrete
- w/p ratio ($<125\ \mu\text{m}$): 0.30–0.35.

This characterization of the composed EMC mixture ignores the w/c ratio of the designed mix. The classical definition of EMC, as it is used for example by Häring [3], considers w/c ratios <0.40 as desirable for EMC. However, such low values of the w/c ratio cannot be achieved by EMC mixes having low cement contents.

Traditional EMC mixes are characterized by low contents of inert or reactive filler materials. Therefore, the difference between the w/c ratio and the w/p ratio is small or non-existent for these mixes when using a classical design approach. Decreasing cement contents and increasing amounts of filler materials, however, augments the difference between the w/c and the w/p ratio. Considering the desired workability of the concrete as a function of the w/p ratio, it is more appropriate for the mix design to take the w/p ratio into account than the w/c ratio. This fact is also reflected by the strength development of the tested mortar samples depicted in Fig. 11a. The mix having a cement content of $175\ \text{kg}/\text{m}^3$ fresh concrete and a w/c ratio of 0.62 (w/p=0.35) achieved higher compressive strength values after 28 days than the mix containing $200\ \text{kg}\ \text{cement}/\text{m}^3$ fresh concrete and the corresponding lower w/c ratio of 0.55 (w/p=0.33). The higher compressive strength of the mix containing less cement is caused by a

better workability due to a slightly higher w/p ratio. The difference in the w/p ratio improved the packing of the mix using constant compaction efforts. Considering the w/c ratio of the two mixes, this is in contrast to the observations made by Locher [30] on cement stone regarding cement hydration and strength development of the hardening cement stone.

According to Locher [30], an increase in the w/c ratio is resulting in higher values of the capillary porosity. Caused by the remaining water content, capillary pores are formed which will be filled with hydration products in a progressed hydration state. As a result of increasing w/c ratios, also the capillary porosity of the cement stone is increasing, which makes the cement stone weak. Therefore, an increase in the w/c ratio will result in a decrease in the compressive strength. This is not affirmed by the conducted experiments as a possible reduction in the compressive strength of the cement stone is compensated by a positive influence on the compressive strength of an optimized and denser granular structure. So, it seems that the absolute water content in a mix is more relevant than the w/c.

Nevertheless, the use of low w/c ratios is still important for the durability properties of the concrete as the capillary pores are influencing the impermeability as well as the durability of the hardened concrete. Therefore, a minimum of capillary pores is aimed for a concrete which is considered as durable [31]. But the application of high contents of fine materials (e.g. stone waste materials) seems to be hindering for this purpose as the water demand and therewith the w/c ratio is increasing with increasing amount of fine materials. Here, the application of plasticizers showed a positive effect. Due to the use of plasticizers, the workability of EMC mixes has been improved for mixes containing high amounts of fine materials and low w/c ratios. Packing fractions between 84 and 85% have been achieved in the lab by using paste content of $0.246\ \text{m}^3/\text{m}^3$ concrete with a w/p ratio of 0.33. These mixes have been resulted in high 28 days compressive strength values between 95.7 and $100.3\ \text{N}/\text{mm}^2$.

Furthermore, the application of plasticizers and stone waste materials in the form of Premix 0–4 showed a positive influence on the water absorption of the produced paving blocks. Based on the preliminary investigations on EMC mixes in the lab, the cement content has been reduced to $250\ \text{kg}/\text{m}^3$ fresh concrete for mixes containing stone waste material. Here, the high content of fine particles in the premixed sand Premix 0–4 (percentage of particles $<125\ \mu\text{m}$ amounts to 11% M/M, see Fig. 5) allows a reduction in the cement content of the mix. Using the Premix 0–4, the cement content can be reduced without changes in the entire PSD of the designed mix since a suitable filler material is provided by this premixed and unwashed aggregate. This allows a more efficient use of the binding material by means of an optimized granular structure and suitable filler materials.

7. Conclusions

It is demonstrated that the packing of the solids is of fundamental importance for the aimed optimization of the properties of earth-moist concrete, both in fresh and hardened state. An optimization in the packing improves the properties of the fresh concrete (water demand, compactibility, green strength) and the properties of the hardened concrete as well.

The new mix design concept can be used for a performance based composition of concrete mixes considering the granular properties of the granular ingredients such as particle size, particle shape and surface texture. Thereby, the improvement of the concrete properties is based upon an aimed optimization of the entire PSD distribution of all solids in the mix. For the optimization of the composed PSD, the geometric packing of continuously graded particles following the modified equation of Andreassen and Andersen according to Funk and Dinger [2] is employed.

The results show that hardened concrete achieves higher compressive strength values if an improved granular structure can be achieved. This includes both an increase in the grain-to-grain points of

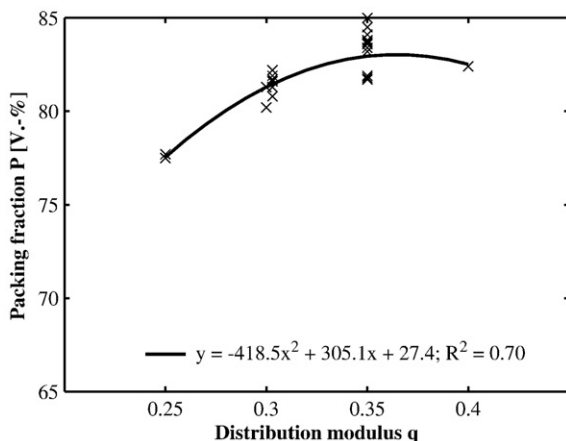


Fig. 13. Influence of various distribution moduli on the packing fraction.

contact and a better packing of aggregates used and results in a more efficient use of the cement clinker.

Considering both the enhancement in the compressive strength as well as the improved cement efficiency, the study reveals how the optimum packing of all concrete ingredients is influencing the mechanical properties positively. Particular attention should not only be paid to high compressive strength values, but also to the fact that a part of the cement seems to be unused if the granular structure of the hardened concrete is very porous and weak due to poor packing of the solids. This causes the addition of cement in order to increase the compressive strength of the hardened concrete. Whereas the added cement is used as a filler material instead of a binding material which contributes primarily to the strength development of the hardened concrete. In view of the market price for cement as well as the energy consumption involved with cement production, this is hardly acceptable for the mass production of concrete products. For the production of cost efficient and environmental friendly concrete products, the cement content should be reduced to a minimum amount necessary for fulfilling the demands on the mechanical and durability properties. This can be achieved by means of an optimum packing and the application of fine stone waste materials as substituent for primary raw materials.

The successful application of stone waste materials in EMC mixes is presented in the second part of this paper. For the successful application of stone waste materials in EMC mixes, the ideas of the new mix design concept are used again. Based on an optimized packing and high contents of fine inert filler materials, cement contents can successfully be reduced down to 250 kg/m³ fresh concrete. Furthermore, the use of superplasticizers showed good results for EMC mixes containing high powder contents. By means of plasticizers, the amount of fine inert particles could be increased without increasing the water content of the mix or a loss in the workability of the concrete mix. Under real production circumstances, this superplasticizer could be omitted as higher compaction efforts are applied on industrial paving block machines for the production of paving blocks.

The new mix design concept showed its suitability for the optimization of the PSD of the composed mix, both for the optimization of already used mixtures and the composition of new mixes containing fine inert filling materials in form of stone waste powders. Moreover, this approach allows for a more performance based mix design of EMC concrete mixes.

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References

- [1] A.H.M. Andreassen, J. Andersen, Ueber die Beziehungen zwischen Kornabstufungen und Zwischenraum in Produkten aus losen Körnern (mit einigen Experimenten), *Kolloid-Zeitschrift* 50 (1930) 217–228 (in German).
- [2] J.E. Funk, D.R. Dinger, *Predictive Process Control of Crowded Particulate Suspensions*, Applied to Ceramic Manufacturing, Kluwer Academic Press, Boston, 1994.
- [3] C. Häring, Development of earth-moist, direct-stripping, mix designs, *Betonwerk + Fertigteil-Technik* 68 (1) (2002) 69–71.
- [4] CEN European Committee for Standardization, EN 1338: Concrete Paving Blocks – Requirements and Test Methods, 2003.
- [5] H.J.H. Brouwers, H.J. Radix, Self-compacting concrete: theoretical and experimental study, *Cement and Concrete Research* 35 (2005) 2116–2136.
- [6] C.C. Furnas, The relation between specific volume, voids, and size composition in systems of broken solids of mixed sizes, Report of Investigation Serial No. 2894, Department of Commerce, Bureau of Mines, 1928.
- [7] H.J.H. Brouwers, Particle-size distribution and packing fraction of geometric random packings, *Physical Review E* 74 (2006) 31309–1–31309-14.
- [8] A. Westman, H. Huggill, The packing of particles, *Journal of the American Ceramic Society* 13 (1930) 767–779.
- [9] C.C. Furnas, Grading aggregates I – mathematical relations for beds of broken solids of maximum density, *Industrial and Engineering Chemistry* 23 (3) (1931) 1052–1058.
- [10] F. De Larrard, T. Sedran, Optimization of ultra-high-performance concrete by the use of a packing model, *Cement and Concrete Research* 24 (6) (1994) 997–1009.
- [11] F. De Larrard, T. Sedran, Mixture-proportioning of high-performance concrete, *Cement and Concrete Research* 32 (11) (2002) 1699–1704.
- [12] M. Mooney, The viscosity of concentrated suspensions of spherical particles, *Journal of Colloids* 6 (1951) 162–170.
- [13] R. Féret, Sur la compacité des mortiers hydrauliques, *Ann. Ponts Chaussée, mémoires et documents, Série 7*, no. IV, 1892, pp. 5–164, (in French).
- [14] W.B. Fuller, S.E. Thompson, The laws of proportioning concrete, *Transactions of the American Society of Civil Engineers* 33 (1907) 222–298.
- [15] A. Hummel, *Das Beton-ABC – Ein Lehrbuch der Technologie des Schwerbetons und des Leichtbetons*, Verlag von Wilhelm Ernst & Sohn, Berlin, 1959 (in German).
- [16] DIN Deutsches Institut für Normung e. V., DIN 1045-2: Concrete, Reinforced and Prestressed Concrete Structures – Part 2: Concrete; Specification, Properties, Production and Conformity; Application Rules for DIN EN 206-1, Beuth Verlag GmbH, Berlin, Germany, 2001 (in German).
- [17] Dutch Normalization-Institute, NEN 5950: Voorschriften Beton Technologie – Eisen, vervaardiging en keuring, Nederlands Normalisatie Instituut, Delft, The Netherlands, 1995 (in Dutch).
- [18] T. Reschke, *Der Einfluss der Granulometrie der Feinstoffe auf die Gefügeentwicklung und die Festigkeit von Beton*, Verlag Bau+Technik GmbH, Düsseldorf, Germany, 2000 (in German).
- [19] M. Hunger, H.J.H. Brouwers, Development of self-compacting eco-concrete, in: H.B. Fischer, F.A. Finger (Eds.), *Proceedings 16th Ibautil, International Conference on Building Materials (Internationale Baustofftagung)*, Weimar, F.A. Finger-Institut für Baustoffkunde, Weimar, Germany, 2006, 2-0189–2-0198.
- [20] M. Schmidt, R. Bornemann, P. Bilgeri, Entwicklung optimierter hüttensandhaltiger Zemente für den Einsatz in der Betonwarenindustrie, *Betonwerk International*, No. 3, 2005 62–72, (in German).
- [21] Bornemann, R., 2005, Untersuchung zur Modellierung des Frisch- und Festbetonverhaltens erdfeuchter Betone. Ph.D. Thesis, University of Kassel, Kassel, Germany (in German).
- [22] Stutech, Aardvochtig Beton. Stutech report No. 22, STUTECH – Studievereniging Betontechnologie, Den Bosch, The Netherlands, 2005 (in Dutch).
- [23] A. Nanni, D. Ludwig, J. Shoenberger, Roller compacted concrete for highway pavements, *Concrete International* 18 (5) (1996) 33–38.
- [24] H. Okamura, M. Ouchi, Self-compacting concrete, *Journal of Advanced Concrete Technology* 1 (1) (2003) 5–15.
- [25] M.R. Jones, L. Zhen, M.D. Newlands, Comparison of particle packing models for proportioning concrete constituents for minimum void ratio, *Materials and Structures* 35 (5) (2002) 301–309.
- [26] DIN Deutsches Institut für Normung e. V., DIN-EN 12350-4: Testing Fresh Concrete – Part 4: Degree of Compactibility; German Version EN 12350-4:1999, Beuth Verlag GmbH, Berlin, Germany, 1999 (in German).
- [27] J. Bonzel, J. Krell, Konsistenzprüfung von Frischbeton, *Beton* 2 (1984) 61–66 (in German).
- [28] DIN Deutsches Institut für Normung e. V., DIN-EN 12390-4: Testing Hardened Concrete – Part 3: Compressive Strength of Test Specimens; German Version EN 12390-4:2000, Beuth Verlag GmbH, Berlin, Germany, 2002 (in German).
- [29] DIN Deutsches Institut für Normung e. V., DIN-EN 12390-6: Testing Hardened Concrete – Part 6: Tensile Splitting Strength of Test Specimens; German Version EN 12390-6:2000, Beuth Verlag GmbH, Berlin, Germany, 2001 (in German).
- [30] F.W. Locher, *Die Festigkeit des Zements*, Beton 8 (1976) 283–286 (in German).
- [31] J. Stark, B. Wicht, *Dauerhaftigkeit von Beton: Der Baustoff als Werkstoff*, Birkhäuser, Basel, 2001 (in German).

Glossary

Roman

a :	Side length [mm]
a_{spl} :	Length of the splitting area [mm]
C_{DIN} :	Degree of compactibility determined according to DIN-EN 12350-4 [25] [–]
C_{rho} :	Degree of compactibility computed according to Eq. (16) [–]
CVC :	Conventionally-vibrated concrete (standard concrete)
D :	Particle size [mm]
D_{max} :	Maximum particle size [mm]
D_{min} :	Minimum particle size [mm]
D_i^{i+1} :	Fraction i
D_{i+1} :	Upper sieve size of fraction D_i^{i+1} [mm]
D_i :	Lower sieve size of fraction D_i^{i+1} [mm]
EMC :	Earth-moist concrete
F :	Force, breaking load [N]
f_c :	Compressive strength according to DIN-EN 12390-4 [27] [N/mm ²]
f_{ct} :	Tensile splitting strength according to DIN-EN 12390-6 [15] [N/mm ²]

f_f :	Flexural strength [N/mm ²]
k :	Correction factor [–]
L_{spl} :	Length related breaking load [N/mm]
M :	Mass [kg]
m :	Number of all ingredients that make up the total concrete mix volume (including air and water)
n :	Number of particle size classes varying in steps of $\sqrt{2}$
$P(D)$:	Computed cumulative finer fraction
$P_{tar}(D_i^{+1})$:	Computed cumulative finer fraction of the target function
$P_{mix}(D_i^{+1})$:	Computed cumulative finer fraction of the composed mixture
Q :	Sieve residue [M.-%]
Q_{mix} :	Computed sieve residue of the composed mixture [M.-%]
PF :	Packing fraction [V.-%]
PSD :	Particle size distribution
q :	Distribution modulus in (modified) Andreasen and Andersen equation [–]
RCC :	Roller-compacted concrete
RCD :	Roller-compacted concrete for dams
RSS :	Sum of the squares of the residuals [–]
R^2 :	Coefficient of determination [–]
s :	Length of the splitting area [mm]
T :	Tensile splitting strength according to EN 1338 [4] [N/mm ²]
t :	Thickness of the paving block [mm]
V :	Volume [m ³]
V_{sol}^{tot} :	Total volume of solids per 1 m ³ fresh concrete [m ³]
V_{cem}^{max} :	Maximum cement volume per 1 m ³ fresh concrete [m ³]
V_{cem}^{min} :	Minimum cement volume per 1 m ³ fresh concrete [m ³]
V_{ves} :	Vessel volume [m ³]
v :	Volumetric amount in concrete mix [–]
x_c :	Compressive strength per kg cement/m ³ concrete [N m ³ /kg mm ²]
x_f :	Flexural strength per kg cement/m ³ concrete [N m ³ /kg mm ²]

Greek

ρ^{den} :	Density densely packed [g/cm ³]
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ρ^{loo} :	Density loosely packed [g/cm ³]
ρ^{spe} :	Specific density [g/cm ³]

Subscript

adm :	Admixture
agg :	Aggregate
air :	Air
c :	Compressive
ct :	Tensile splitting
cem :	Cement
con :	Concrete
f :	Flexural
fil :	Filler
i :	Fraction index, index of sieve size
k :	Counter variable for materials
mix :	Composed mixture
pow :	Powder
pas :	Paste
sol :	Solid
spl :	Splitting
tar :	Target function
ves :	Vessel
wat :	Water

Superscript

den :	Densely packed
loo :	Loosely packed
max :	Maximum
min :	Minimum
spe :	Specific
tot :	Total



Erratum

Erratum to “A new mix design concept for earth-moist concrete: A theoretical and experimental study” [Cement and Concrete Research 38(10) 2008 1246–1259]

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In Cement and Concrete Research 38, 1246–1259 (2008), A new mix design concept for earth-moist concrete: A theoretical and experimental study, by G. Hüsken and H.J.H. Brouwers, the caption of Fig. 4a and b should read:

Fig. 4a: Sizes and definition of fractions used in the optimization algorithm. b: Composed aggregate mix; $D_{\max} = 16 \text{ mm}$, $D_{\min} = 0.275 \mu\text{m}$, $q = 0.35$.

Furthermore, Eqs. (5), (10) and (14) should be replaced by

$$R^2 = 1 - \frac{\sum_{i=1}^n (P_{\text{mix}}(D_i^{i+1}) - P_{\text{tar}}(D_i^{i+1}))^2}{\sum_{i=1}^n (P_{\text{mix}}(D_i^{i+1}) - \bar{P}_{\text{mix}})^2} \quad \forall D_i^{i+1} \in [D_{\min}, D_{\max}], \quad (5)$$

$$v_{\text{sol},k} \geq 0 \quad \text{for } k = 1, 2, \dots, m-2, \quad (10)$$

and

$$w/p = \frac{M_{\text{wat}}}{\sum_{k=1}^{m-2} M_{\text{sol},k}} = \frac{\rho_{\text{wat}} V_{\text{wat}}}{\sum_{k=1}^{m-2} \rho_{\text{sol},k}^{\text{spe}} V_{\text{sol},k}(D_i)} \quad \text{for } D_i < 125 \mu\text{m}, \quad (14)$$

respectively.

The first explanation of Eq. (8) should read:

with $Q_{\text{sol},k}(D_i)$ sieve residue of material k on sieve i .

The typological errors have no further impact on the content of the paper.

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