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The effect of blowing or suction on laminar free convective heat transfer on flat horizontal plates

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Abstract. In the present paper laminar free convective heat transfer on flat permeable horizontal plates is investigated. To assess the effect of surface suction or injection on heat transfer a correction factor, provided by the film model (or "film theory"), is applied. Comparing the film model predictions with numerical results of previous boundary layer analyses yields good agreement for a wide range of dimensionless transpiration levels.

Der Einfluß von Ausblasung oder Absaugung auf den Wärmeübergang an ebenen horizontalen Platten bei laminarer freier Konvektion

Zusammenfassung: In der Arbeit wird der Wärmeübergang an ebene stoffdurchlässige horizontale Platten bei laminarer freier Konvektion untersucht. Zur Abschätzung des Einflusses von Absaugung oder Ausblasung auf den Wärmeübergang wird ein aus der Filmtheorie entnommener Korrekturfaktor herangezogen. Ein Vergleich der Voraussagen nach dem Filmmodell mit vorliegenden numerischen Ergebnissen nach dem Grenzschichtmodell ergab gute Übereinstimmungen in einem weiten Bereich des dimensionslosen Transpirationsgrades.

Nomenclature

C	dimensionless mass flux
c_p	specific heat [J kg^{-1} K^{-1}]
E	relative difference between literature and film model, Eq. (13)
f_w	dimensionless mass flux
Gr_x	Grashof number, $g\beta(T_w - T_\infty)x^3/\nu^2$
g	gravitational acceleration [ms^{-2}]
h_x	local heat transfer coefficient, Eq. (4) [Wm^{-2} K^{-1}]
K	constants
k	thermal conductivity [Wm^{-1} K^{-1}]
L	plate length [m]
Nu_x	local Nusselt number, Eq. (8)
n	power-law coefficient
Pr	Prandtl number, $\nu c_p \varrho/k$
q	heat flux [Wm^{-2}]
T	temperature [K]
v	component of velocity in the y-direction [ms^{-1}]
x	coordinate along the plate [m]
y	coordinate normal to the plate [m]

Greek symbols

β	coefficient of thermal expansion [K^{-1}]
η	dimensionless similarity variable, Eq. (6)
Θ_t	ratio of actual heat transfer and impermeable plate heat transfer
Θ	dimensionless temperature, Eq. (5)
ν	kinematic viscosity [m^2 s^{-1}]
ϱ	density [kg m^{-3}]
ϕ_t	dimensionless wall mass flux, Eq. (3)

Subscripts

film	pertaining to film model
num	pertaining to numerical solution
w	wall
∞	ambient fluid

Superscripts

*	pertaining to impermeable plate
′	differentiation with respect to η
–	mean

1 Introduction

Free convection flows over horizontal flat plates are encountered in many practical applications, for instance heat transfer from/to ceilings or floors. The flow along and heat transfer to impermeable plates have been studied by Stewartson [1], Gill et al. [2], Rotem and Claassen [3], Goldstein et al. [4], Jones [5], Pera and Gebhart [6] and Umemura and Law [7]. The effect of surface injection or suction have been studied by Clarke and Riley [8, 9], Afzal and Hussain [10], Lin and Yu [11] and Ramanaiah et al. [12].

The effect of wall transpiration on heat transfer can also be assessed with the help of the classical film model. This model provides analytical expressions, the so-called correction factors, which can be applied to any system of importance. In the past these film model correction factors have been successfully used to predict the effect of surface mass transfer on transport phenomena, such as heat transfer. The correction factors can be derived analytically from a simple stagnant film analysis and applied to systems using either an imposed (transpiration) wall mass flux or a diffusional vapour mass flux (by condensation or evaporation). Recent reviews of the film model are found in Bannwart [13], Bannwart and Bontemps [14], Brouwers [15] and Brouwers and Chesters [16].

To the author's knowledge, the film model has never been applied to free convective heat transfer between a permeable horizontal wall and a fluid. In the present analysis the film theory correction for the effect of wall transpiration on heat transfer, the so-called Ackermann factor, is applied to this problem. Subsequently, the predictions of the compact film model are thoroughly compared with the numerical suction results of Hussain [17], who proceeded from the physical model presented in Afzal and Hussain [10], and suction and injection results of Ramanaiah et al. [12].

2 Film model

According to the film theory the actual local heat transfer, denoted by q_w, in the presence of mass transfer follows from multiplying the zero suction (or neutral) q_w^* by a correction factor:

$$q_w = \Theta_{t,\text{film}}\, q_w^*. \tag{1}$$

The thermal correction factor, commonly referred to as Ackermann correction, follows from e.g. [16] as:

$$\Theta_{t,\text{film}} = \frac{-\phi_t}{e^{-\phi_t} - 1}, \tag{2}$$

where the dimensionless mass flux towards the wall reads:

$$\phi_t = -\frac{v_w \varrho c_p}{h_x^*}. \tag{3}$$

In this equation h_x^* represents the local heat transfer coefficient in the case of zero mass transfer which is defined as:

$$h_x^* = \frac{q_w^*}{T_\infty - T_w} = \frac{k \left.\dfrac{dT}{dy}\right|_{y=0}^*}{T_\infty - T_w}, \tag{4}$$

where y is a coordinate normal to the plate.

Using

$$\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{5}$$

$$\eta = \frac{y}{x} Gr_x^{1/5}, \tag{6}$$

Eq. (4) yields:

$$h_x^* = -\frac{k\,\Theta'(0)^*}{x} Gr_x^{1/5}, \tag{7}$$

where the prime represents differentiation with respect to η. Following [1, 2] $\Theta'(0)$ amounts to -0.3574 for free convective heat transfer between an isothermal horizontal plate and a fluid with $Pr = 0.72$. If the local Nusselt number is defined as:

$$Nu_x = \frac{x h_x}{k}, \tag{8}$$

one obtains as neutral Nusselt number correlation:

$$Nu_x^* = 0.3574\, Gr_x^{1/5}. \tag{9}$$

It is interesting to note that for free convective heat transfer to a vertical isothermal plate Ostrach [18] obtained:

$$Nu_x^* = 0.5046\, Gr_x^{1/4}, \tag{10}$$

for $Pr = 0.72$.

From Eqs. (1)–(3) and Eq. (7) it can readily be concluded that, following the film model, the effect of a surface mass flux on heat transfer can be assessed in a relatively simple way.

3 Comparison with previous work

The film model correction for the effect of mass transfer on heat transfer is compared with the results of [10, 12, 17]. References [10, 17] concern a boundary layer analysis of laminar free convective flow over a horizontal isothermal plate with imposed suction. Ramanaiah et al. [12] present a study of both suction and injection.

References [10, 12, 17] considered power-law variations of the wall temperature ($T_w - T_\infty = K_1 x^n$) and the transpiration velocity ($v_w = K_2 x^{(n-2)/5}$) under which self-similar solutions of the governing equations are possible. For various dimensionless wall suction parameters, denoted by C in [10, 17] and by f_w in [12], the dimensionless temperature gradient at the wall, $-\Theta'(0)$, were determined numerically. The heat transfer ratio now readily follows as:

$$\frac{q_w}{q_w^*} = \frac{\Theta'(0)}{\Theta'(0)^*} = \Theta_{t,\text{num}}, \tag{11}$$

which can be compared with the film model correction factor. To this end, the dimensionless mass flux ϕ_t has to be assessed. This property is related to C by:

$$\phi_t = -\frac{C(n+3)Pr}{5\,\Theta'(0)^*}, \tag{12}$$

see Eqs. (3) and (7), and equation (36) from [10]. For $n = 0$ and $Pr = 0.72$ values of $\Theta'(0)$ were computed by [17], using the model of [10], as a function of various C. These values are summarized in Table 1. A glance at this table shows that $\Theta'(0)^*$ amounts to -0.3571. This value is in agreement with the value provided by [1, 2], mentioned in the previous section. Furthermore, it is also in accordance with the value provided in [3]; $\Theta'(0)^*$ (referred to as "$H'(0)$") equals -0.35909 there.

Table 1 includes both the pertaining film model correction factors and the relative error defined as:

$$E = \frac{\Theta_{t,\text{film}} - \Theta_{t,\text{num}}}{\Theta_{t,\text{num}}}. \tag{13}$$

The tabulated values of E indicate that for suction, i.e. $C > 0$ and $\phi_t > 0$, the film model predictions well correspond to the

Table 1. The effect of wall suction on heat transfer according to Hussain [17] and the film model (T_w is constant)

Hussain [17]		Eq. (11)	Eq. (12)	Eq. (2)	Eq. (13)
C	$-\Theta'(0)$	$\Theta_{t,\mathrm{num}}$	ϕ_t	$\Theta_{t,\mathrm{film}}$	E
0.0	0.3571	1.0000	0.0000	1.0000	0.0%
1.0	0.6186	1.7323	1.2097	1.7239	−0.5%
2.0	0.9380	2.6267	2.4195	2.6558	−1.1%
3.0	1.3220	3.7020	3.6292	3.7281	−0.7%
4.0	1.7376	4.8659	4.8390	4.8390	−0.5%

Table 2. The effect of wall transpiration on heat transfer according to Ramanaiah et al. [12] and the film model (T_w is constant)

Ramanaiah et al. [12]		Eq. (11)	Eq. (16)	Eq. (2)	Eq. (13)
f_w	$-\Theta'(0)$	$\Theta_{t,\mathrm{num}}$	ϕ_t	$\Theta_{t,\mathrm{film}}$	E
−1.0	0.1786	0.4997	−1.2087	0.5145	3.0%
0.0	0.3574	1.0000	0.0000	1.0000	0.0%
1.0	0.6121	1.7126	1.2087	1.7232	0.6%

Table 3. The effect of wall transpiration on heat transfer according to Ramanaiah et al. [12] and the film model (q_w is constant)

Ramanaiah et al. [12]		Eq. (11)	Eq. (16)	Eq. (2)	Eq. (13)
f_w	$-\Theta'(0)$	$\Theta_{t,\mathrm{num}}$	ϕ_t	$\Theta_{t,\mathrm{film}}$	E
−1.0	0.2749	0.5959	−1.0405	0.5684	−4.6%
0.0	0.4613	1.0000	0.0000	1.0000	0.0%
1.0	0.7201	1.5610	1.0405	1.6089	3.1%

Table 4. The effect of wall transpiration on heat transfer according to Ramanaiah et al. [12] and the film model (v_w is constant)

Ramanaiah et al. [12]		Eq. (11)	Eq. (16)	Eq. (2)	Eq. (13)
f_w	$-\Theta'(0)$	$\Theta_{t,\mathrm{num}}$	ϕ_t	$\Theta_{t,\mathrm{film}}$	E
−1.0	0.4894	0.6498	−0.9560	0.5970	−8.1%
0.0	0.7531	1.0000	0.0000	1.0000	0.0%
1.0	1.0995	1.4600	0.9560	1.5530	6.4%

numerical results. For larger suction rates the deviation initially increases, but subsequently the film model and numerical predictions converge again. In the following it is explained that the film model prediction and asymptotic solution (as derived in [10]) for large suction rates are equivalent indeed.

Combining equations (22) and (24) of [10] yields:

$$q_w = \left.\frac{dT}{dy}\right|_{y=0} = \frac{(T_w - T_\infty)\,Pr}{\nu}, \tag{14}$$

as expression for the heat transfer in the case of large suction. Eqs. (3), (4) and (14) produce:

$$q_w = \phi_t\, q_w^*. \tag{15}$$

For large suction Afzal and Hussain [10] thus obtained this asymptotic relation between actual (permeable plate) heat flux and neutral (impermeable plate) heat flux. A glance at the film model correction factor in Eq. (2) shows that Θ_t also tends to ϕ_t for large ϕ_t. So the fact that the results of [10] and the film model converge for very large suction rates is in fact no surprise.

From Eqs. (3), (7), and equation (16) of [12] one can relate ϕ_t and f_w by:

$$\phi_t = -\frac{f_w(n+3)\,Pr}{5\,\Theta'(0)^*}. \tag{16}$$

For $Pr = 0.72$, and $n = 0$ (constant T_w), $n = 1/3$ (constant q_w) and $n = 2$ (constant v_w) values of $\Theta'(0)$ were computed for $f_w = -1, 0$ and 1. The numerical results of [12] and the predictions of the film model are presented in Tables 2–4.

The negative f_w of these tables corresponds to injection, while positive f_w corresponds to suction. Comparing Tables 1 and 2 reveals that the impermeable plate results ($C = f_w = 0$) and suction results ($C = f_w = 1$) of [10, 17] are in agreement with those of [12]. Furthermore, in all cases Tables 1–4 illustrate the substantial effect of suction/injection on heat transfer, as well the good agreement between film model and numerical similarity solutions.

For purely forced convective heat transfer along a permeable plate Mickley et al. [19] obtained a comparable agreement between the boundary layer analysis, the film model and performed experiments. As the film model provides adequate predictions for both purely free and purely forced convective heat transfer, it is expected that the predictions are reliable for the intermediate case of mixed convection along a horizontal permeable plate as well.

4 Mean heat transfer

In the foregoing the attention has been focused on the effect of mass transfer on local heat transfer coefficients. In what follows the effect on mean heat transfer coefficients is analyzed in some detail.

The mean neutral heat transfer coefficient is defined as:

$$\bar{q} = \frac{1}{L}\int_{x=0}^{L} q_w\, dx. \tag{17}$$

In case of no mass transfer, substitution of Eqs. (4) and (7) transforms Eq. (17):

$$\bar{q}^* = -\frac{5\,k\,\Theta'(0)^*}{3\,L}\,Gr_L^{1/5}\,(T_\infty - T_w). \tag{18}$$

For convection with mass transfer, i.e. $\Theta_{t,\mathrm{film}} \neq 1$, the integral of Eq. (17) can be solved in closed form too, since ϕ_t and hence $\Theta_{t,\mathrm{film}}$, are constant, see Eqs. (1)–(2) and (12). This feature of constant ϕ_t (and C or f_w) is a result of the imposed similarity solution. Hence, the mean heat transfer then simply follows from multiplying the mean neutral heat transfer, governed by Eq. (18), by the – constant – film model correction factor.

5 Conclusions

In this paper the film model has been applied to free convective heat transfer in the presence of mass transfer. The film model correction factor for heat transfer has been extensively compared with existing theoretical results of Afzal and Hussain [10, 17] and Ramanaiah et al. [12]. These elaborations involved a laminar free convective boundary layer flow over a porous horizontal plate. In these studies the governing equations were derived and solved numerically by imposing a similarity solution, the Prandtl number being set equal to 0.72.

For a wide range of transpiration levels the film model appeared to agree within a few per cent with the results of the literature. Furthermore, it was proved that for infinitely large suction the film model correctly predicts the asymptotic behaviour of the process. Hence, one can conclude that the basic film model is well suited to describe the effect of transpiration on free convective heat transfer to horizontal flat permeable plates.

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