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A method for calculating equivalent diameter of fiber in Self-Compacting Fiber Reinforced Concrete

Abstract:

This paper presents a method for calculating the equivalent diameter of fiber in self-compacting fiber reinforced concrete (SCFRC). The key idea is to utilize a small amount of particles with a narrow particle size distribution to replace the fibers by the same volume, without causing any obvious changes of the properties of concrete in both fresh and hardened states. Some test methods, such as slump-flow, V-funnel, air content and compressive strength, were employed to evaluate the properties change of the concrete. The results show that the equivalent diameter of the fiber (L_f/d_f=13 mm/0.2 mm, L_f and d_f means the length and diameter of fiber respectively) is 2.2 mm, which means the fiber can be treated as a type of spherical aggregate with the diameter of 2.2 mm in the application of particle packing-based concrete design models.

Introduction

Self-compacting concrete (also called self-consolidating concrete and abbreviated as SCC) was first produced in Japan in 1993 [1]. Due to its high flowability and rheological stability, SCC is treated as a milestone in concrete technology and is widely applied to cast floors, elements with complicated shape and concentrated reinforcement or in prefab industry. It is generally accepted that the addition of fibers can improve some mechanical properties of brittle cementitious materials, but also decreases their workability [2-5]. Hence, an appropriate combination of SCC and fibers to produce the self-compacting fiber reinforced concrete (SCFRC) with a sufficient flowability and mechanical properties in fresh and hardened state respectively can bring significant influence on the application of SCFRC.

A considerable amount of work has been devoted over the last decade to investigate the suitable design method of SCC. De Larrard [6] utilized the “compressive packing model” to design the SCC. In the research of Saak et al. [7], a new segregation-controlled design methodology is introduced for SCC, which suggests that aggregate segregation is governed by the yield stress, viscosity and density of the cement paste matrix. Based on the paste rheology criteria, Bui et al. [8] proposed a rheological model for SCC, which includes the minimum apparent viscosity, minimum flow and optimum flow-viscosity ratio to achieve SCC with satisfactory segregation resistance and deformability. According to the Andreasen and Andersen model [9] and the modified Andreasen and Andersen model [10], Brouwers and Radix [11] demonstrated that applying the continuous geometrical packing theory, the particles can be better packed, which results in improved hardened state properties as well as an improved workability, since more water is available to act as lubricant between the granular particles. However, due to the contradiction between the fiber shape and the assumptions of the spherical particles taken into account in most of the models that have already been utilized to design SCC, it is not an easy work to include the fiber into these models.

As for the inclusion of the fiber into the mix design, Yu et al. [12, 13] proposed a concept called the “equivalent packing diameter” with an assumption that replacing the non-spherical particle by a fictitious sphere does not result in the change of the packing density. De Larrard [6] proposed a method to include stiff steel fibers into the “compressive packing model” with the definition of “perturbed zone”. Ferrara et al. [14] proposed a concept of “equivalent spherical particle” (assuming that the sphere and fiber should have the same specific surface area and mass) and included the fibers into the modified Andreasen and Andersen model. Nevertheless, some contradictory results can be obtained from using different models for calculating the equivalent diameter of the same fiber, and therefore more attention needs to be paid to this topic.

The objective of the research presented in this paper is to propose a new method to calculate the equivalent diameter of fiber. The key idea is to utilize a small amount of granular particles with a narrow particle size distribution to replace the fibers by the same volume, without bringing any significant changes of the properties of concrete in both fresh and hardened states. Laboratory test results of SCC and SCFRC are reported and discussed in the paper.

Raw materials

The raw materials applied in this study are listed as follows: the Portland cement used was CEM I 42.5N (ENCI); the coarse aggregates used were composed of broken granite in fraction of 2-8 mm; two different sands were used: dredged river sand (0-4 mm) and microsand (0-1 mm); ground limestone powder was applied as filler; a polycarboxylate based superplasticizer was used to guarantee sufficient followability of concrete. In addition, straight steel fiber (L_f/d_f=13 mm/0.2 mm) and some particles for replacement (termed R-particles) with the size range of 1.0-1.4 mm, 1.4-2.0 mm, 2.0-2.8 mm, 2.8-4.0 mm and 4.0-5.6 mm (the first four fractions were obtained by sieving the dredged sand (0-4 mm) and the last fraction was obtained by sieving the broken granite (2-8 mm)) were also employed in this research.

Mix design of SCC and SCFRC

In this study, the modified Andreasen and Andersen model is utilized to design the concrete mixtures, which is illustrated as follows [10]:

$$P(D) = \frac{D^q - D_{\min}^q}{D_{\max}^q - D_{\min}^q}$$

(1)

where $P(D)$ gives the cumulative passing fraction at a sieve with opening D, D_{max} is the maximum particle size (μm) and the D_{min} is the minimum particle size (μm), q is distribution modulus. The proportions of the individual particle materials in the concrete are adjusted until an optimum fit between the composed mix and the target curve is reached [15-16]. Based on the research of Hunger [17], the used value of q is 0.23. The optimized mixture proportions of SCC and SCFRC are shown in Table 1. An example (mixture A) of the target line and the resulting integral grading line of the mix are shown in Fig. 1.

Table 1
Mixture proportions (kg/m^3) of SCC and SCFRC

Mixtures	Cement	Limestone	Sand (0-4)	Micro-sand	Granite (2-8)	R-particles	Fiber	SP	Water
A,B,C,D,E✦	420.0	167.7	606.5	181.8	752.2	13.2	0	2.52	193.2
SCFRC	420.0	167.7	606.5	181.8	752.2	0	39	2.52	193.2

(✦ where A, B, C, D and E represent mixtures containing the R-particles ranging from 1.0-1.4 mm, 1.4-2.0 mm, 2.0-2.8 mm, 2.8-4.0 mm and 4.0-5.6 mm respectively.)

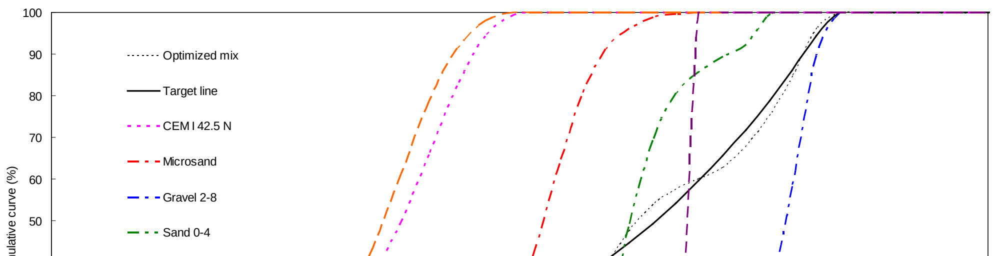


Fig. 1
The target line and the resulting integral grading line of mix A

Fresh and hardened state behaviour tests of concrete

According to the European Guidelines for Self-Compacting Concrete, the fresh state behaviour of concrete was tested by slump-flow and V-funnel tests [18]. Furthermore, for the testing of the air content in fresh concrete, a model is proposed as follows:

$$M_c = M_{solid} + M_{liquid} \tag{2}$$

where M_c is the mass of the total concrete in a container of a known volume (8 dm^3), M_{solid} is the mass of the solid particles in the container and M_{liquid} is the mass of the liquid in the container. Additionally:

$$V_c = V_{solid} + V_{liquid} + V_{air} \tag{3}$$

where V_c is the volume of the container, V_{solid} is the volume of solid particles in the container, V_{liquid} is the volume of liquid in the container and V_{air} is the volume of air in the container. Hence, the air content in SCC and SCFRC can be calculated from the following equation:

$$V_{air} = V_c - \left(\sum_i \frac{M_i}{\rho_i} + \sum_j \frac{M_j}{\rho_j} \right) \tag{4}$$

where M_i and ρ_i are the mass and density of the fraction i in solid materials, and M_j and ρ_j are the mass and density of the fraction j in liquid materials respectively. In addition, the packing fraction (PF) of solid in each mixture can also be calculated easily as follows:

$$PF = V_{solid} / V_c = (V_c - V_{liquid} - V_{air}) / V_c \tag{5}$$

For each mixture, 150 mm cubes were cast, demoulded after 24 hours and subsequently cured in water. Then, the compressive strength was measured at 7 days.

Review of the available methods for calculating the equivalent diameter of fibers

Based on the assumption that replacing a non-spherical particle with a fictitious sphere does not result in the change of the packing density, Yu [12-13] proposed a model to calculate the equivalent diameter of cylindrical particles as follows:

$$d_p = \left(3.1787 - 3.6821 \times \frac{1}{\varphi} + 1.5040 \times \frac{1}{\varphi^2} \right) \times d_v$$

(6)

where d_P is the equivalent packing diameter, d_V is the volume diameter (diameter of a sphere having the same volume as the particle) and φ is the sphericity (ratio of the surface area of the sphere having the same volume as the particle, to its actual surface area). The values of d_V and φ can be obtained as follows:

$$d_v = 1.145 \times \sqrt[3]{L_f / d_f} \times d_f \quad , \quad \varphi = 2.624 \times \frac{\left(\sqrt[3]{(L_f / d_f)} \right)^2}{1 + 2 \times (L_f / d_f)}$$

(7)

where: d_f is the fiber diameter (mm) and L_f is the fiber length (mm).

Moreover, under the assumption that the surface area of the total amount of fibers added to a unit volume of concrete has to correspond to the surface area of an equal mass of spheres having the same (average) specific weight of aggregates, another model was proposed by Ferrara [14] as follows:

$$d_{eq-fiber} = \frac{3L_f}{1 + 2 \times \frac{L_f}{d_f}} \times \frac{\gamma_{fiber}}{\gamma_{aggregate}}$$

(8)

where L_f and d_f is the length and diameter of the fibers respectively, γ_{fiber} is the specific weight of fibers and $\gamma_{aggregate}$ is the weighted average specific weight of all the aggregates.

The new method proposed in this study

From the investigation of Bui [8], the average spacing between aggregate particles of SCC can be calculated using the following equation:

$$D_{ss} = D_{av} \times \left(\sqrt[3]{1 + \frac{V_p - V_{void}}{V_c - V_p}} - 1 \right)$$

(9) where D_{ss} is average spacing between aggregate particle surfaces, V_p is paste volume, V_{void} is the volume of voids in densely compacted aggregates, V_c is total concrete volume and D_{av} is the average aggregate diameter, which is given by:

$$D_{av} = \sum d_i \times m_i \Big/ \sum m_i$$

(10)

where D_{av} is the average aggregate diameter, d_i is average size of aggregate fraction *i* and m_i is the percentage of aggregate mass retained between upper and lower sieve sizes (obtained from sieve analysis) in fraction *i*. Hence, for the fiber-reinforced skeleton, the "average equivalent diameter of solid particles" ($D_{av-SCFRC}$) can be expressed as [14]:

$$D_{av-SCFRC} = \frac{\sum d_i m_i + d_{eq-fiber} \times m_{fiber}}{\sum m_i + m_{fiber}}$$

(11)

where d_i is average size of aggregate fraction *i*, m_i is the percentage of aggregate mass retained between upper and lower sieve sizes (obtained from sieve analysis) in fraction *i*,
$d_{eq-fiber}$ is the equivalent diameter of fiber and m_{fiber} is the mass of the fibers.

In this study, the following assumption is proposed: the D_{ss} value will remain the same when the fibers of SCFRC are volumetrically replaced by the R-particles without causing any significant changes of properties of concrete. Hence, the equivalent diameter of fiber can be expressed as follows:

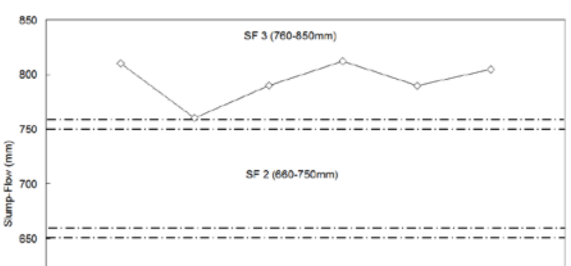
$$d_{eq-fiber} = \frac{\left[\frac{\sum d_i' m_i'}{\sum m_i'} \times \left(\sqrt[3]{\left(1 + \frac{V_p' - V_{void}'}{V_c' - V_p'} \right)} - 1 \right) \Big/ \left(\sqrt[3]{\left(1 + \frac{V_p - V_{void}}{V_c - V_p} \right)} - 1 \right) \right] \times \left(\sum m_i + m_{fiber} \right) - \sum d_i m_i}{m_{fiber}}$$

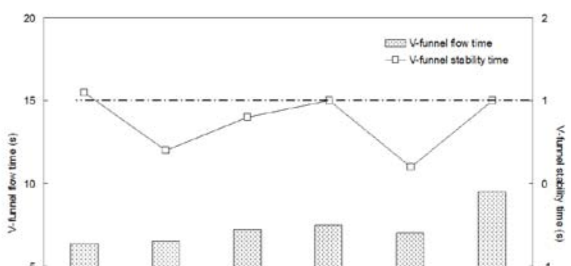
(12)

where d_i, m_i, V_p, V_{void}, V_c are obtained for SCFRC and d_i', m_i', V_p', V_{void}', V_c' are obtained for SCC (the one that has the most similar properties as that of SCFRC).

Fresh state concrete

Fig, 2 presents the results of slump-flow, V-funnel and air content of tested concrete. It can be noticed clearly from Fig. 2a that all the mixtures are classified into SF 3 flowability class of SCC, which means the slump-flow value of each mixture is higher than 760 mm. In addition, mixtures A (1.0 - 1.4 mm) and D (2.8 - 4.0 mm) show similar values of slump-flow as that of SCFRC, which is about 800 mm. As can be seen in Fig. 2b, the V-funnel flow time of SCFRC is obviously higher than for the other mixtures, which means that the addition of fibers increases the viscosity of concrete effectively. In addition, the 5-minute stability time of mixtures A (1.0 - 1.4 mm), C (2.0 - 2.8 mm) and D (2.8 -4.0 mm) are all fluctuating around 1s, which is the same as that of SCFRC. In Fig. 2c, it can be found that the air content and packing fraction of mixture C (2.0 - 2.8 mm) and D-(2.8 - 4.0 mm) are more similar as that of SCFRC. Hence, utilizing the particles ranging from 2.0-2.8 mm or the ones ranging from 2.8-4.0 mm to replace fibers by the same volume in the concrete will not bring any significant changes in the particle packing. In summary, based on all the results obtained on fresh concrete, mixture C (2.0 - 2.8 mm) and D (2.8 - 4.0 mm) can be treated as the two that have the most similar properties as that of mixture SCFRC.

a)


b)


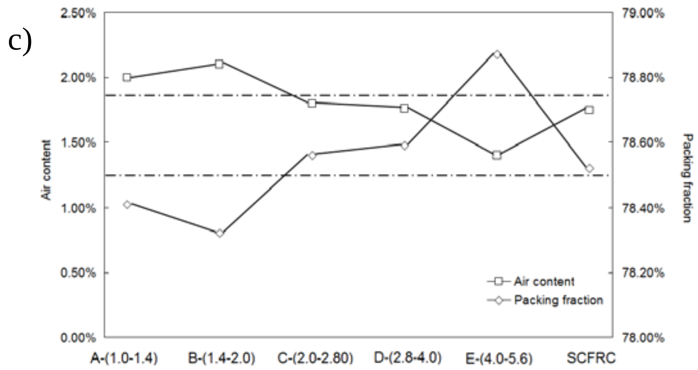


Fig.2
Results of test on concrete in fresh state, a) slump-flow, b) V-funnel, c) air content and packing fraction

Hardened state concrete

Fig. 3 illustrates the compressive strength at 7 days measured for each mixture and its deviation from the SCFRC. It appears that the strength of all the mixtures are very comparable, about 55 MPa. However, the deviation from the SCFRC is smallest for mixture C (2.0 - 2.8 mm) and D (2.8 - 4.0 mm), which is 2.3% and 2.8% respectively. Consequently, the results of hardened concrete show that the mixture C and D can be treated as the potential mixtures that have the most similar properties to that of SCFRC.

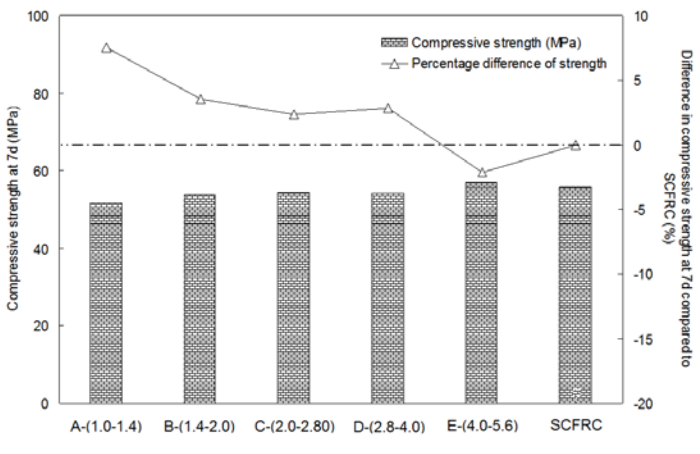


Fig.3
7 days compressive strength test results of hardened concrete

Calculation results of the equivalent diameter of fiber

The model from Yu (Eq. (6) and (7)) and Ferrara (Eq. (8)) suggest that the equivalent diameter of straight steel fiber (L_f/d_f=13 mm/0.2 mm) is 5.6 mm and 0.9

mm, respectively. According to the experiments performed in this study, it can be found that these predictions are not accurate enough.

Here, according to the experimental test results, the mixtures C (2.0 - 2.8 mm) and D (2.8 - 4.0 mm) were chose as the ones having the most similar properties as that of SCFRC, and were utilized to calculate the equivalent diameter of fiber that used in the study. From Eq. (12) and properties of mixtures C and D, the equivalent diameter of fiber is 2.2 mm and 2.5 mm, respectively. It is worth to notice that the calculated result for mixture D (2.8 - 4.0 mm) is 2.5 mm, which is not in the range of 2.8 - 4.0 mm, but in the size range of mixture C. Therefore, the results show that the mixture C (2.0 - 2.8 mm) is more suitable to replace the mixture of SCFRC and should be utilized to calculate the equivalent diameter of fiber theoretically.

To sum up, the equivalent diameter of fiber proposed here should be 2.2 mm, which means the investigated fiber (L_f/d_f=13 mm/0.2 mm) can be treated as spherical particles with the diameter of 2.2 mm in the design of SCFRC.

Conclusions

This paper proposes an experimental method for calculating equivalent diameter of fiber in self-compacting fiber reinforced concrete (SCFRC). Based on the obtained results, the equivalent diameter of the employed fiber (L_f/d_f=13 mm/0.2 mm) is 2.2 mm, which means this fiber can be treated as a type of spherical aggregate with the diameter of 2.2 mm in the concrete design. However, only one single type of fiber was investigated in this study, and therefore more research is still needed to evaluate the efficiency and accuracy of this method in the future research.

Acknowledgments

The authors wish to express their gratitude to the following sponsors of the Building Materials research group at TU Eindhoven: Rijkswaterstaat Centre for Infrastructure, Graniet-Import Benelux, Kijlstra Betonmortel, Struyk Verwo, Attero, Enci, Provincie Overijssel, Rijkswaterstaat Directie Zeeland, A&G Maasvlakte, BTE, Alvon Bouwsystemen, V.d. Bosch Beton, Selor, Twee "R" Recycling, GMB, Schenk Concrete Consultancy, Intron, Geochem Research, Icopal, BN International, APP All Remove, Consensor, Eltomation, Knauf Gips, Hess ACC Systems and Kronos (in chronological order of joining).

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