LETTERS TO THE EDITOR

To the Editor:

A revision of the Ackermann (1937) correction factor for heat transfer accompanied by mass transfer was proposed recently by Brouwers (1995). This revision was proposed for situations in which the thermal film thickness exceeds the diffusional film thickness, i.e., for $\delta_T > \delta_{AB}$ in the notation of Bird et al. (1960).

The development by Brouwers (1995) considers the solute molar flux N_{Ay} (relative to the interface) to be zero outside the diffusional film, i.e., for $y > \delta_{AB}$, though nonzero inside the film. This assumption is incompatible with steady-state mass balances across the boundary $y = \delta_{AB}$ or any region containing that boundary, in the asserted presence of mass transfer to or from the main stream.

This contradiction can be avoided by using mass balances to evaluate the species fluxes as functions of y, as was done in each of the previous works listed below. One finds the species fluxes N_{iy} in a nonreacting fluid to be independent of y according to the one-dimensional film model, rather than stepping abruptly to zero at $y = \delta_{AB}$ for any transferred species.

The resulting expressions for interfacial fluxes, corrected for mass transfer of any or all species, are given by Mickley et al. (1954) and by Bird et al. (1960). These results include those of Ackermann (1937), and are preferable to the results of Brouwers (1995).

Film models, with their drastic assumptions, are inappropriate for detailed calculations of transport near interfaces. Better predictions of interphase transfer and related phenomena are obtainable via boundary layer methods, on which there is an extensive literature. Boundary-layer solutions for the Ackermann problem and its diffusional analog are included in Mickley et al. (1954) and in Bird et al. (1960), for example.

Literature cited

Ackermann, G., "Wärmeübertragung und Molekulare Stoffübertragung im Gleichen Feld bei Grossen Temperaturr und Partial-Druckdifferenzen," *VDI Forschungsheft*, **382**, 1 (1937).

Bird, R. B., W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley, New York, Chaps. 19 and 21 (1960).

Brouwers, H. J. H., "Stagnant Film Model for Effect of Diffusional Layer Thickness on Heat Transfer and Exerted Friction," *AIChE J.*, **41**, 1821 (1995).

Mickley, H. S., R. C. Ross, A. L. Squyers, and W. E. Stewart, "Heat, Mass and Momentum Transfer for Flow over a Flat Plate with Blowing or Suction," *NACA Tech. Note*, No. 3208 (1954).

Warren E. Stewart
Dept. of Chemical Engineering
University of Wisconsin
Madison, WI 53706

Reply:

For lateral transport in the main stream (bulk), i.e., perpendicular to the wall, a lateral gradient is required. In the bulk, however, velocity, temperature, and composition are constant so that no lateral transport takes place. This feature was used by Brouwers (1995) in deriving the improved film model: beyond the diffusional film there is no mass transfer, and the effect of this reduced film thickness on both heat and momentum transfer has been accounted for. This approach results in correction factors for heat transfer and exerted friction that tend to be unity if the mass-transfer layer reduces to zero, which would be expected physically. According to the reasoning of Prof. Stewart, the correction factors would not depend at all on the magnitude of the diffusional layer thickness, which is not the case.

Furthermore, there is another impediment for the fluxes to be present in the bulk, i.e., beyond their respective films. Though the film model is based on a stagnant film, its results are also used for flow in closed channels. For symmetry, at the center of a channel the fluxes obviously have to equal zero. This contention also makes the continuation of the fluxes in the bulk not arguable.

It is interesting to realize that the film model represents a simplification of the boundary layer theory, whereby the film model thickness of mass, heat and momentum transfer are directly related to their respective boundary layer thicknesses. In the boundary layer theory, velocity, composition and temperature are also constant beyond their boundary layers, and no momentum, mass and heat are transferred in the bulk. In fact, in the film model by Brouwers (1995), various film thicknesses are accounted for in a similar way as in the boundary layer theory. For instance, see the problem treated by Schlichting (1987) where the thermal and momentum boundary layers are of different magnitude depending on the Prandtl number.

Summarizing, the proposed revision of the film model by Brouwers (1995) is allowed and its results recommended for future computations.

Literature cited

Brouwers, H. J. H., "Stagnant Film Model for the Effect of Diffusional Layer Thickness on Heat Transfer and Exerted Friction," *AIChE J.*, **41**, 1821 (1995).

Schlichting, H., *Boundary Layer Theory*, 7th ed., McGraw-Hill, New York, p. 289 (1987).

H. J. H. Brouwers
Dept. of Civil Engineering and Management
University of Twente
7500 AE Enschede, The Netherlands

Rejoinder:

This rejoinder is prompted by Dr. Brouwers' reply. The references cited are given in the above letter.

In film models of nonreactive systems, lateral transport beyond the film is demanded by the conservation of energy and of each chemical species. Lateral transport does indeed require lateral gradients, but these are traditionally suppressed by assuming large eddy diffusivities, α_{eff} and $\mathfrak{D}_{\text{eff}}$, in the outer stream. The predicted lateral gradients of temperature and composition are of order $1/\alpha_{\text{eff}}$ and $1/\mathfrak{D}_{\text{eff}}$, respectively, and thus can be rendered very small though never identically zero. The conduction and diffusion fluxes remain of order α_{eff}^0 and $\mathfrak{D}_{\text{eff}}^0$, so they cannot be neglected outside the film, nor can the corresponding convective fluxes.

Forbidding heat and mass transport beyond the film, as Brouwers (1995) did, gives a useless model which forbids heat and mass exchange altogether! This assertion is readily verified by macroscopic balances across the film or by using Brouwers' zero-flux conditions at the outer boundary (as he forgot to do) when integrating his one-dimensional conservation equations across the films. This fatal flaw negates the arguments in Brouwers' first paragraph, all of which presume the validity of his model.

For axisymmetric flows in tubes, the radial fluxes vary strongly with r and vanish on the axis regardless of their values elsewhere. A similar remark applies to flows with a symmetry plane. Thus, Brouwers' symmetry argument for closed-channel flows applies only locally and does not imply that the lateral fluxes vanish at other main-stream locations.

Brouwers' attempt to justify his film model via boundary layer theory is unfounded. Film models, being one-dimensional, demand different boundary conditions. Regions of negligible diffusion appear in boundary layer theory because of phenomena neglected in film models, such as two-dimensional convection with inflow of fresh main-stream fluid. Flux-free regions cannot occur in one-dimensional, steady-state, nonreacting systems, because material or energy which enters such a system has no escape except through the opposite boundary.

Brouwers finds these features of standard film models dissatisfying. The writer shares this view. But the remedy is not to be found in any modified one-dimensional model. Rather, one should use a model with at least two space dimensions, so that convection can be properly described and the distributions of state variables can be reliably predicted. Boundary layer provides such a model.

The film analysis proposed by Brouwers (1995) should not be used. The treatment by Ackermann (1937), extended in Bird et al. (1960), is better. Boundary-layer theory is preferable to either of these approaches; some early results at large net mass fluxes are given in Bird et al. (1960).

W. E. Stewart
Dept. of Chemical Engineering
University of Wisconsin
Madison, WI 53706